AEDC-TR-66-16

EQUATIONS AND CHARTS FOR THE EVALUATION OF FORCES ON SPHERICALLY BLUNTED CONES BY THE NEWTONIAN THEORY

L. L. Trimmer

ARO, Inc.

April 1966

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FOREWORD

The work reported herein was done at the request of Headquarters, Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC) under Program Element 65402234.

The results of research presented were obtained by ARO, Inc. (a subsidiary of Sverdrup and Parcel, Inc.), contract operator of AEDC, AFSC, Arnold Air Force Station, Tennessee, under Contract AF40(600)-1200. The research was conducted under ARO Project No. VT3116, and the manuscript was submitted for publication on December 22, 1965.

The author would like to acknowledge the assistance of Mrs. P. Trenchi, Mr. E. L. Clark, and Mr. W. R. Martindale in the preparation of this report.

This technical report has been reviewed and is approved.

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ABSTRACT

Equations and charts are presented for the determination of the aerodynamic forces and moments of spherically blunted cones in hypersonic flow by the modified Newtonian theory. The equations are valid for cone half-angles from 0 to 90 deg and angles of attack from 0 to 180 deg; charts are presented for cone half-angles from 5 to 40 deg and angles of attack from 0 to 90 deg. A comparison of Newtonian theory predictions with experimental data is presented.

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NOMENCLATURE

C_A	Axial-force coefficient
C_m	Pitching-moment coefficient
C_N	Normal-force coefficient
C_p	Pressure coefficient
K	Proportionality constant used in modified Newtonian theory
ℓ	Moment coefficient reference length
M_∞	Free-stream Mach number
R_b	Base radius of cone
R_n	Spherical nose radius
S	Reference area
$\overline{X}$	Correlation parameter, see Fig. 5b
X_{cp}	Distance from base of cone to center of pressure (positive forward)
X_t	Transfer distance from base of cone to moment reference point (positive forward)
α	Angle of attack
γ	Ratio of specific heats of test gas
δ	Cone half-angle
η	Angle between a line normal to body surface and free-stream velocity
ξ	Cone bluntness ratio, R_n/R_b

SECTION I
INTRODUCTION

In the analysis of aerodynamic data and the conduction of wind tunnel tests, it is frequently desirable to provide a theoretical estimate of the aerodynamic loading of a particular configuration. In the hypersonic flow regime the Newtonian theory has proved useful for these applications because of its relative simplicity. Closed form analytic expressions for the Newtonian aerodynamic coefficients of several basic shapes are presented in Ref. 1. Two of these shapes, the spherical nose and the cone frustum, are the components of the spherically blunted cone (Fig. 1), which has received considerable attention in the realm of hypersonic aerodynamics.

The equations presented in Ref. 1 for the spherical nose and the cone frustum have been combined to provide expressions for the aerodynamic coefficients of the spherically blunted cone. The equations are valid for cone half-angles from 0 to 90 deg and angles of attack from 0 to 180 deg. To facilitate the use of these equations, they have been numerically evaluated and are presented graphically for a range of cone half-angles from 5 to 40 deg and angles of attack from 0 to 90 deg.

SECTION II
DISCUSSION

The Newtonian impact theory provides a simple relationship between the surface pressure coefficient and the local body inclination to a hypersonic free-stream flow. Adaptation of Newton's original work to these parameters gives the familiar form of the Newtonian equation:

$$C_p = 2 \cos^2 \eta \tag{1}$$

In more recent work, however, the constant 2 has been replaced with an arbitrary constant K. This form of the equation has been selected for this work, and the choice of a numerical value for the constant has been left to the user. There are several suggested means of determining the constant - each having merit for particular applications. For example, good results are usually obtained for slender bodies with attached shocks by using either the Newtonian value of 2 or with the following equation:

$$K = \frac{C_{p_{nose}}}{\sin^2 \delta_{nose}} \tag{2}$$

where $C_{p_{nose}}$ is determined from exact cone values such as those presented in Ref. 2. For blunt bodies, better results may be obtained by using the maximum pressure coefficient ($C_{p_{max}}$) for the constant. With Mach numbers greater than 6 this quantity is closely approximated by

$$C_{p_{max}} = \frac{\gamma + 3}{\gamma + 1} \tag{3}$$

which for air (γ = 1.4) becomes 1.83.

The geometry of the spherically blunted cone and associated nomenclature are shown in Fig. 1. In order to retain as much generality as possible in the derivation of the coefficients, the choice of reference area and length is left arbitrary. Frequently the base area and base diameter are used for S and ℓ, respectively.

2.1 NORMAL FORCE

The equations for the normal-force coefficients of the basic body components, the spherical cap and the cone frustum, as given in Ref. 1, have been combined and are presented below. Because of limitations in the geometrical description of the surfaces as the upper portions become shielded from the flow, the equations are separated into two angle ranges. It should be noted that for any portion of the surface which is shielded from the flow, i.e. when η is greater than $\pi/2$, the assumption is made that the pressure coefficient is zero.

For $0 \leq \alpha \leq \delta$

$$\frac{C_N S}{K R_b^2} = \pi \sin\alpha \cos\alpha \cos^2\delta \left(1 - \frac{\xi^2}{2}\cos^2\delta\right) \tag{4}$$

For $\delta \leq \alpha \leq (\pi - \delta)$

$$\frac{C_N S}{K R_b^2} = \sin\alpha\cos\alpha\cos^2\delta\left(1 - \frac{\xi^2}{2}\cos^2\delta\right)\left[\frac{\pi}{2} + \sin^{-1}\left(\frac{\tan\delta}{\tan\alpha}\right)\right]$$

$$+ \frac{\xi^2}{6}\sin\alpha\left[3\cos^{-1}\left(\frac{\sin\delta}{\sin\alpha}\right) + \sin\delta\left(5 - \frac{2}{\sin^2\alpha} + \frac{\sin^2\delta}{\sin^2\alpha} - 3\sin^2\delta - \frac{4}{\sin^2\delta}\right)\left(\sin^2\alpha - \sin^2\delta\right)^{1/2}\right]$$

$$+ \left(\frac{2\sin\alpha\cos^2\delta}{3\sin\delta} + \frac{\cos^2\alpha\sin\delta}{3\sin\alpha}\right)\left(\sin^2\alpha - \sin^2\delta\right)^{1/2} \tag{5}$$

These equations have been evaluated and are presented in chart form as a function of angle of attack with constant values of ξ and δ in Fig. 2.

2.2 AXIAL FORCE

Similarly the following equations for axial-force coefficient have been evaluated and are shown in chart form in Fig. 3.

For $0 \leq \alpha \leq \delta$

$$\frac{C_A S}{K R_b^2} = \frac{\pi}{2}\left[\left(1 - \frac{\xi^2}{2}\cos^2\delta\right)\left(2\cos^2\alpha\sin^2\delta + \sin^2\alpha\cos^2\delta\right) + \xi^2\cos^2\alpha\cos^2\delta\right] \tag{6}$$

For $\delta \leq \alpha \leq (\pi - \delta)$

$$\frac{C_A S}{K R_b^2} = \left[\frac{\pi}{2} + \sin^{-1}\left(\frac{\tan\delta}{\tan\alpha}\right)\right]\left\{\cos^2\alpha\left[\frac{\xi^2}{2} + \sin^2\delta\left(1 - \xi^2\cos^2\delta\right) - \frac{\xi^2}{2}\sin^4\delta\right] + \frac{\sin^2\alpha\cos^2\delta}{2}\left(1 - \frac{\xi^2}{2}\cos^2\delta\right)\right\}$$

$$+ \frac{\cos\alpha}{2}\left\{\left(\sin^2\alpha - \sin^2\delta\right)^{1/2}\left[\frac{3\xi^2}{2}\sin^3\delta + \sin\delta\left(3 - \frac{5\xi^2}{2}\right)\right] + \xi^2\cos^{-1}\left(\frac{\sin\delta}{\sin\alpha}\right)\right\} \tag{7}$$

Application of Eq. (7) for angles of attack greater than 90 deg will require consideration of the base contribution. This can be easily computed with the following equation and added algebraically to the results obtained from Eq. (7).

For $\pi/2 \leq \alpha \leq \pi$

$$\left(\frac{C_A S}{K R_b^2}\right)_{base} = -\pi\cos^2\alpha \tag{8}$$

2.3 CENTER OF PRESSURE

The center-of-pressure locations, measured from the base, are shown in Fig. 4 and were obtained by dividing the pitching moment by the normal force. Equations for this quantity are:

For $0 \leq \alpha \leq \delta$

$$\frac{X_{cp}}{R_b} = \frac{\cos\delta - \frac{2}{3\cos\delta} + \frac{1}{6}\xi^2\cos^2\delta(\xi - 3\cos\delta)}{\sin\delta\left(1 - \frac{\xi^2}{2}\cos^2\delta\right)} \tag{9}$$

Independent of α

For $\delta \leq \alpha \leq (\pi - \delta)$

$$\frac{X_{cp}}{R_b} = \frac{KR_b^3}{C_N S}\left[\frac{\sin\alpha\cos\alpha}{\tan\delta}\left[\frac{\pi}{2} + \sin^{-1}\left(\frac{\tan\delta}{\tan\alpha}\right)\right]\left[\frac{1}{6}\xi^3\cos^3\delta - \frac{1}{2}\xi^2\cos^4\delta + \cos^2\delta - \frac{2}{3}\right]\right.$$
$$+ \left(\sin^2\alpha - \sin^2\delta\right)^{1/2}\left\{\frac{\xi^2}{6}\sin\alpha\sin\delta\left[\frac{1-\xi\cos\delta}{\tan\delta} - \xi\sin\delta\right]\left[3\sin^2\delta - \frac{\sin^2\delta}{\sin^2\alpha} - 5\right]\right.$$
$$\left. + \left[\cos^2\delta\left(1 - \xi^2\cos^2\delta\right) - \frac{2}{3}\left(1 - \xi^3\cos^3\delta\right)\right]\left[\frac{2\sin^2\alpha\cos^2\delta + \cos^2\alpha\sin^2\delta}{3\sin\alpha\sin^2\delta\cos\delta}\right]\right\}$$
$$\left. + \cos^{-1}\left(\frac{\sin\delta}{\sin\alpha}\right)\left[\frac{\xi^2}{2}\sin\alpha\left(\frac{1-\xi\cos\delta}{\tan\delta} - \xi\sin\delta\right)\right]\right] \quad (10)$$

where C_N is obtained from Eq. (5).

The most useful application of the center of pressure is to calculate the pitching-moment coefficient,

$$C_m = C_N\left(\frac{X_{cp} - X_t}{\ell}\right) \quad (11)$$

where X_t is the distance from the base to the desired reference point.

2.4 EXPERIMENTAL CORRELATION

Correlation of experimental data from several blunted cones has been accomplished with considerable success by Whitfield and Wolny (Ref. 3). The correlation curves given in Ref. 3 have been fitted through experimental data. These semi-empirical correlation curves offer a convenient means of comparing specific Newtonian calculations with experimental data within, of course, the range of parameters considered in the original correlation (α = 0 to 30 deg, δ = 6.3 to 20 deg, ξ = 0 to 0.5, and M_∞ = 8 to 21.7). Deviations of the experimental data from the correlation curves were ±0.04 for C_N and ±0.10 for C_m. A comparison of Newtonian calculations and the correlation curves of Ref. 3 is presented in Fig. 5. A value of K = 2 was used for the normal-force coefficient calculations. It should be noted that the value of K does not enter into the correlation of pitching-moment coefficient because of the inclusion of C_N in the correlation parameter.

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3. Whitfield, Jack D. and Wolny, W. "Hypersonic Static Stability of Blunt Slender Cones." AEDC-TDR-62-166 (AD282897), August 1962.

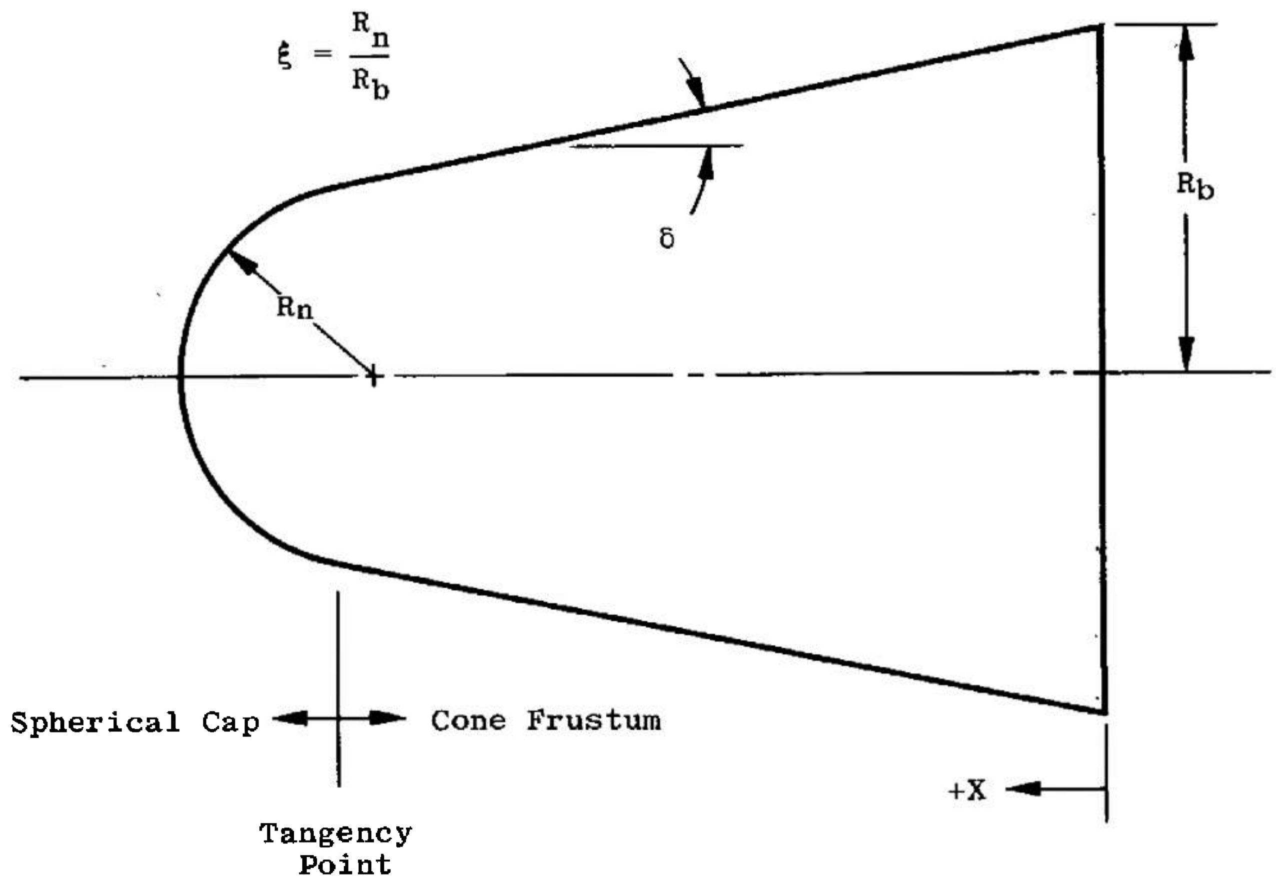
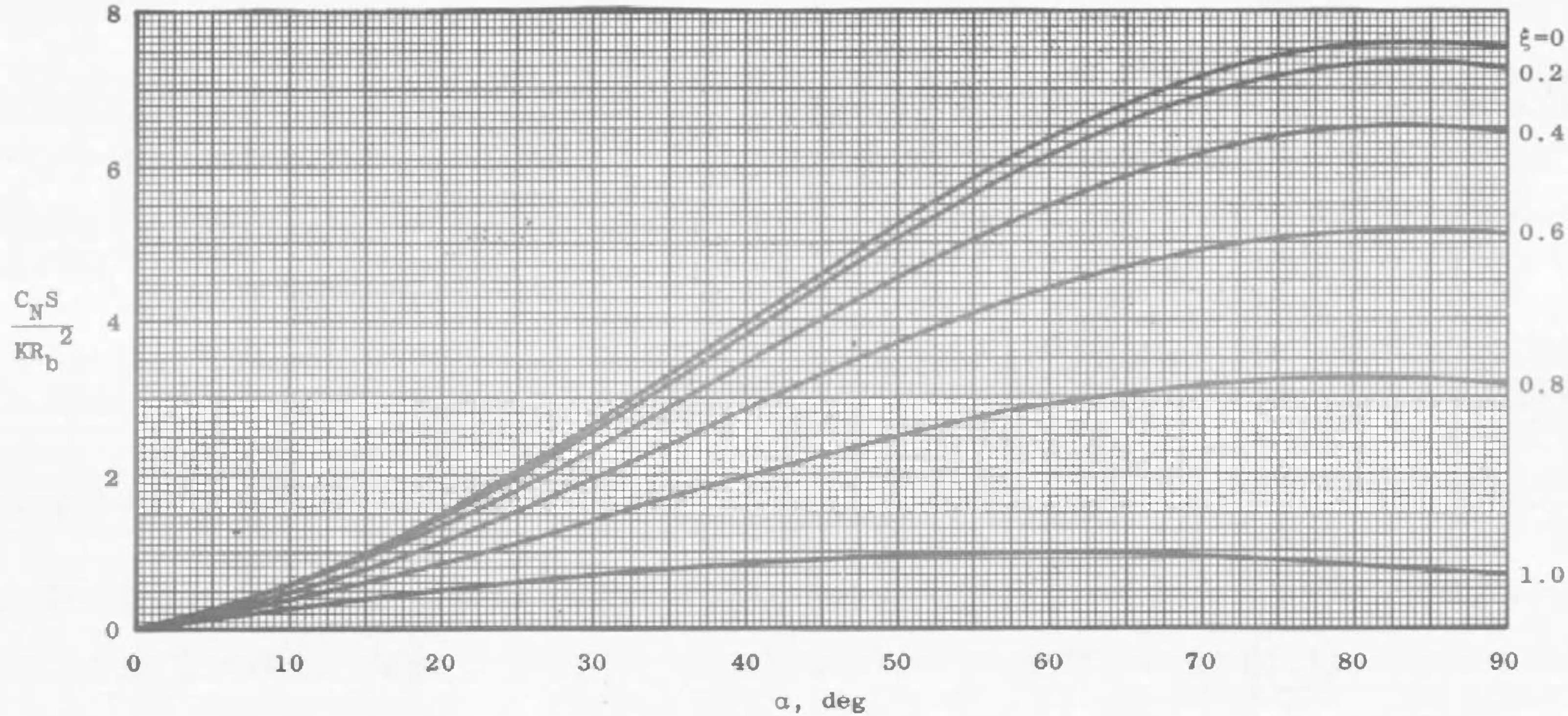
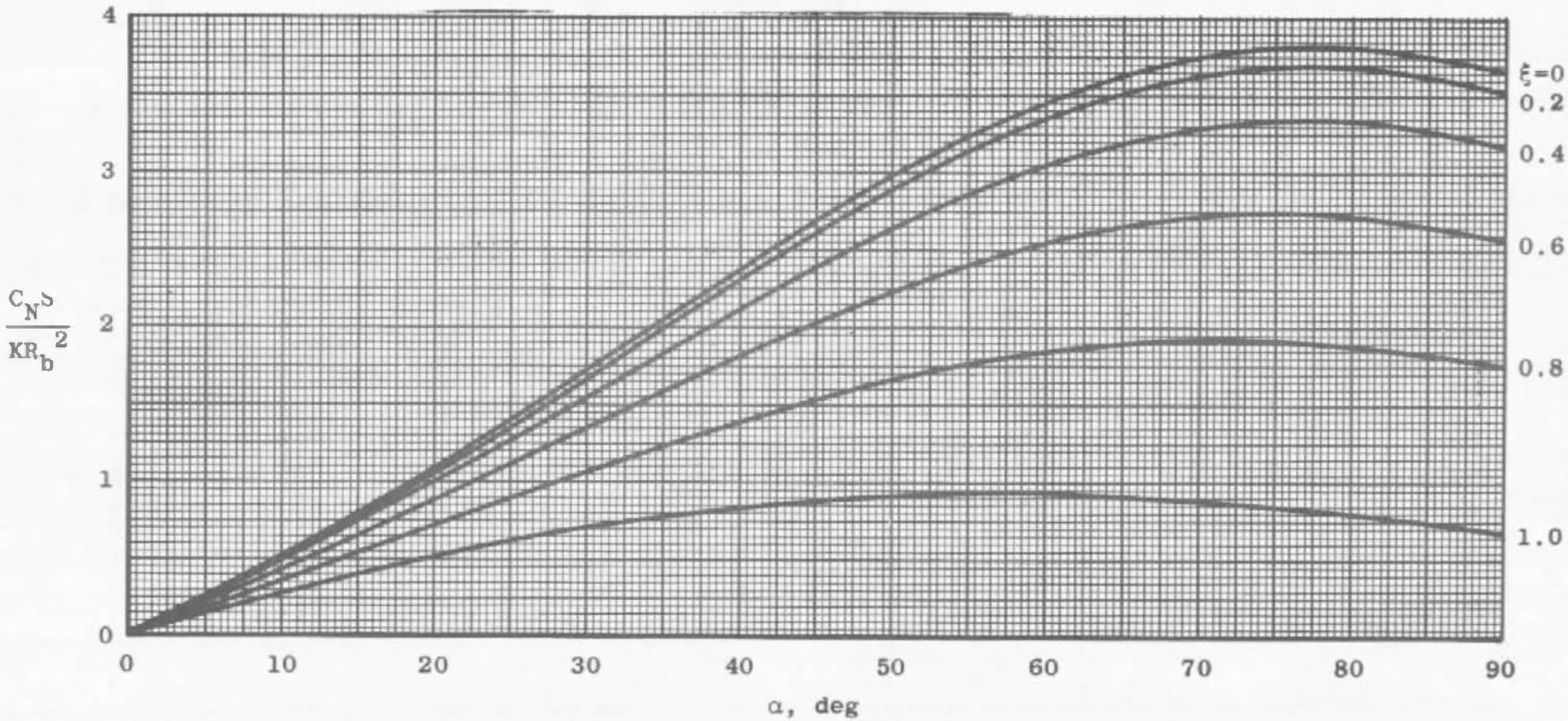


Fig. 1 Spherically Blunted Cone Geometry and Nomenclature



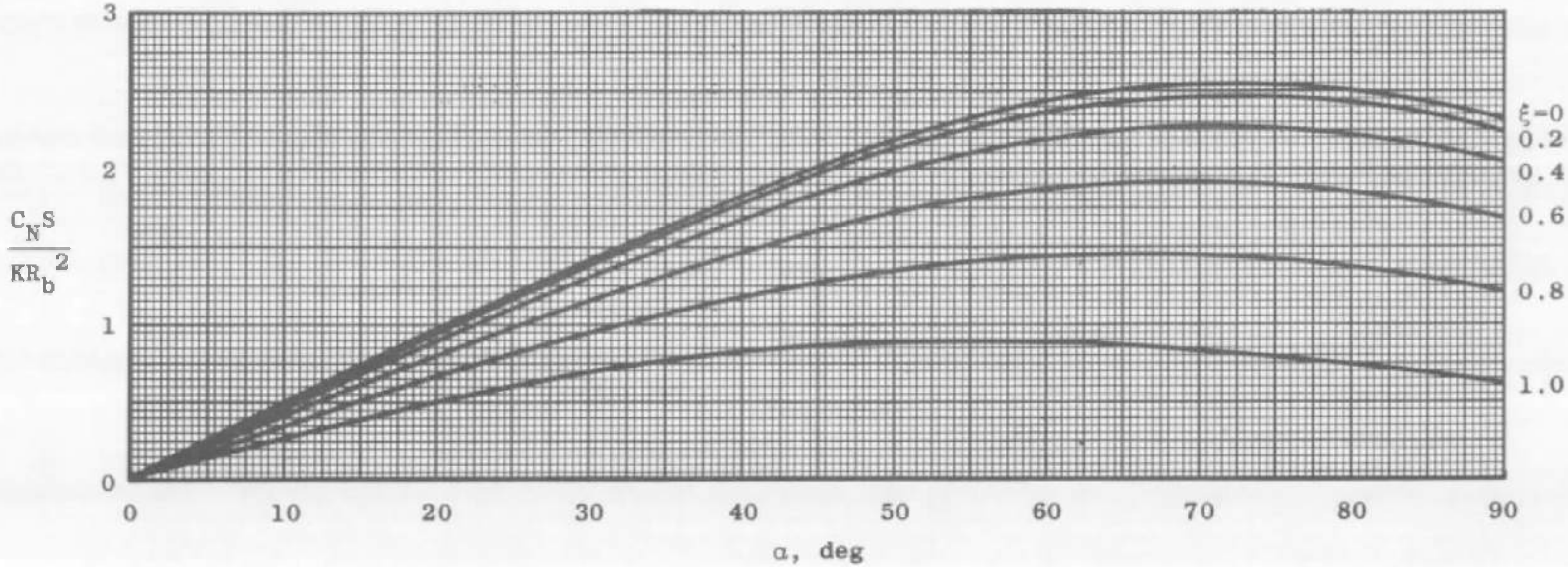
a. δ = 5 deg

Fig. 2 Normal-Force Coefficient



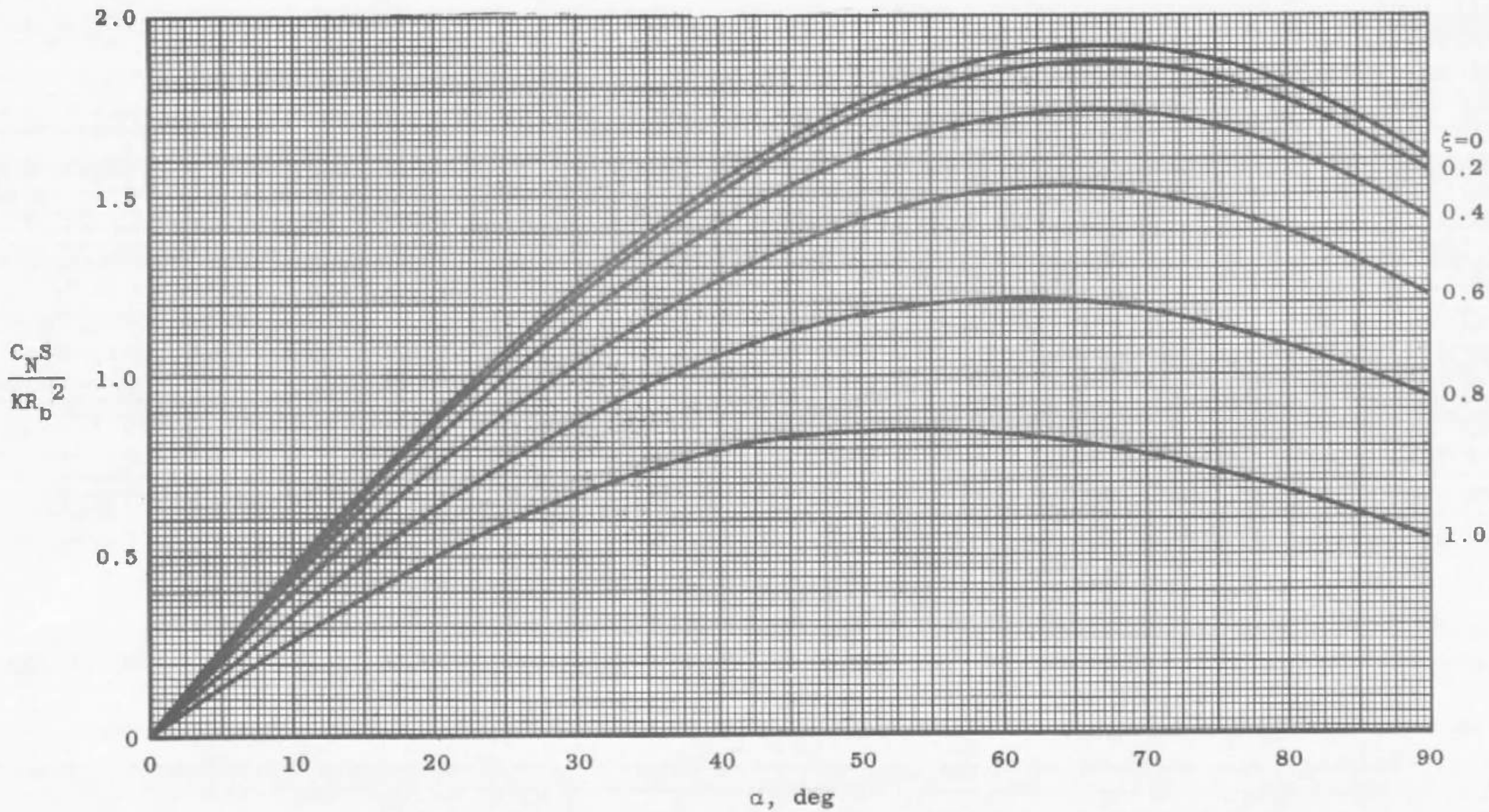
b. $\delta = 10$ deg

Fig. 2 Continued



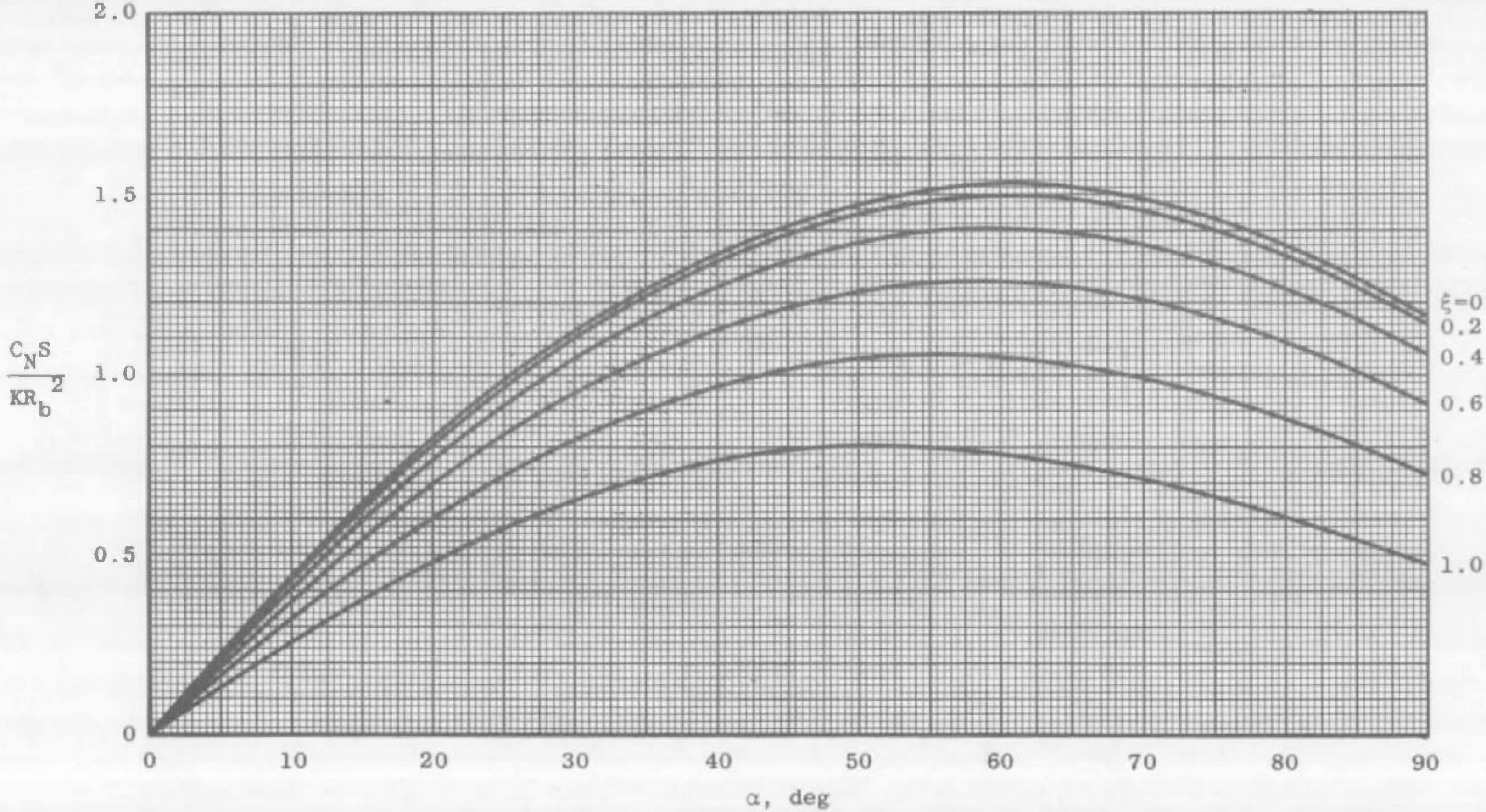
c. δ = 15 deg

Fig. 2 Continued



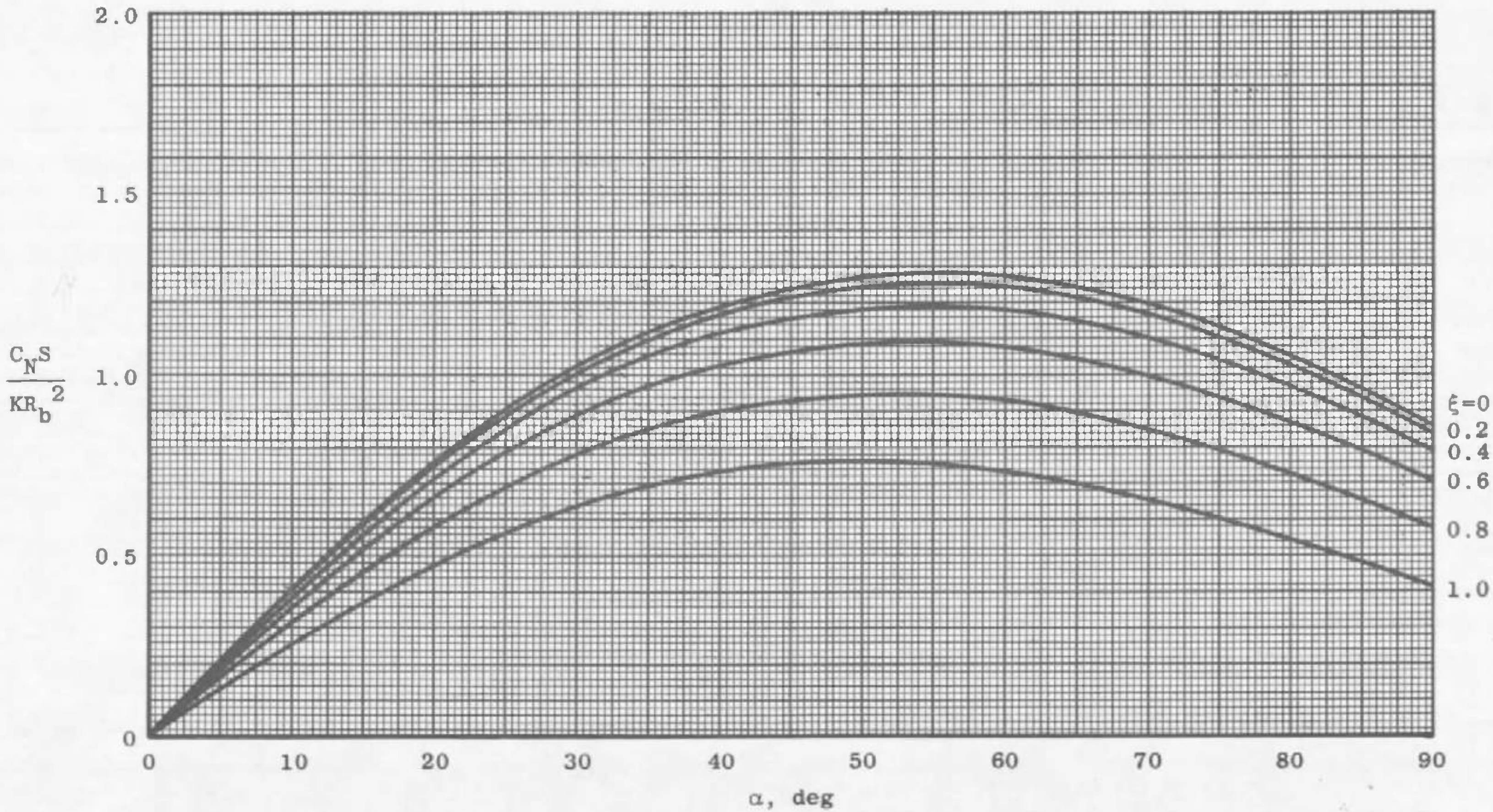
d. δ = 20 deg

Fig. 2 Continued



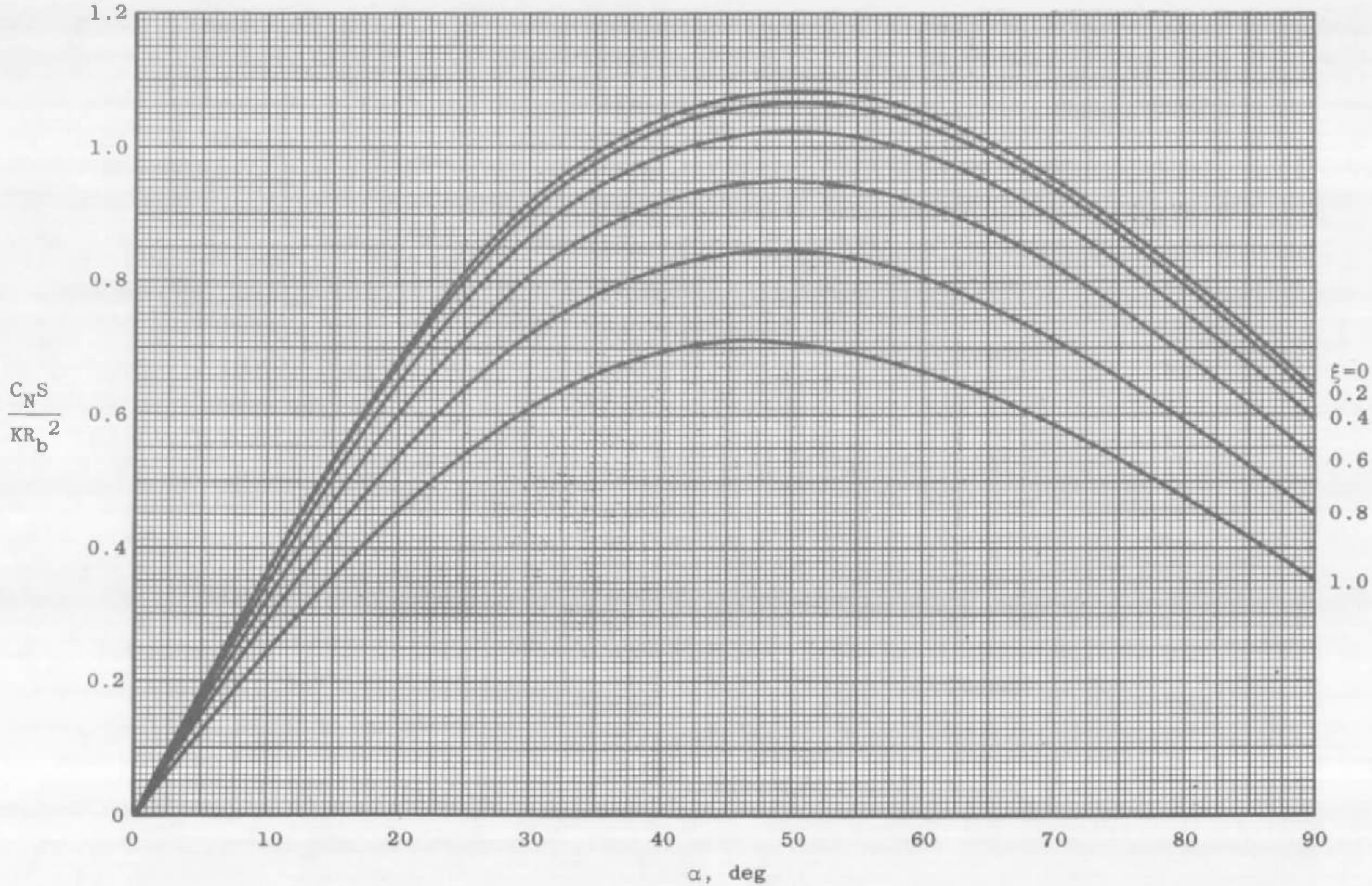
e. δ = 25 deg

Fig. 2 Continued



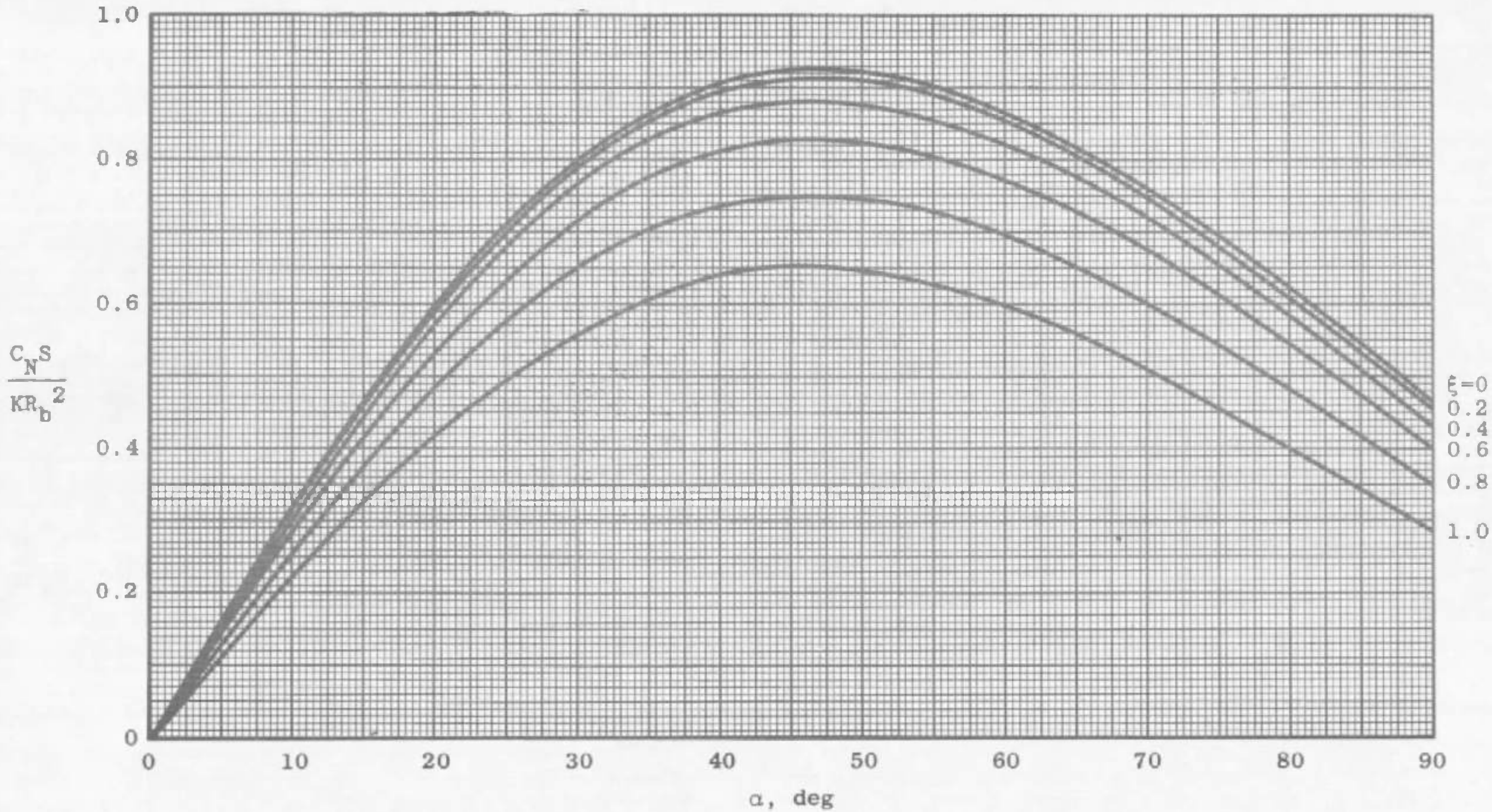
f. δ = 30 deg

Fig. 2 Continued



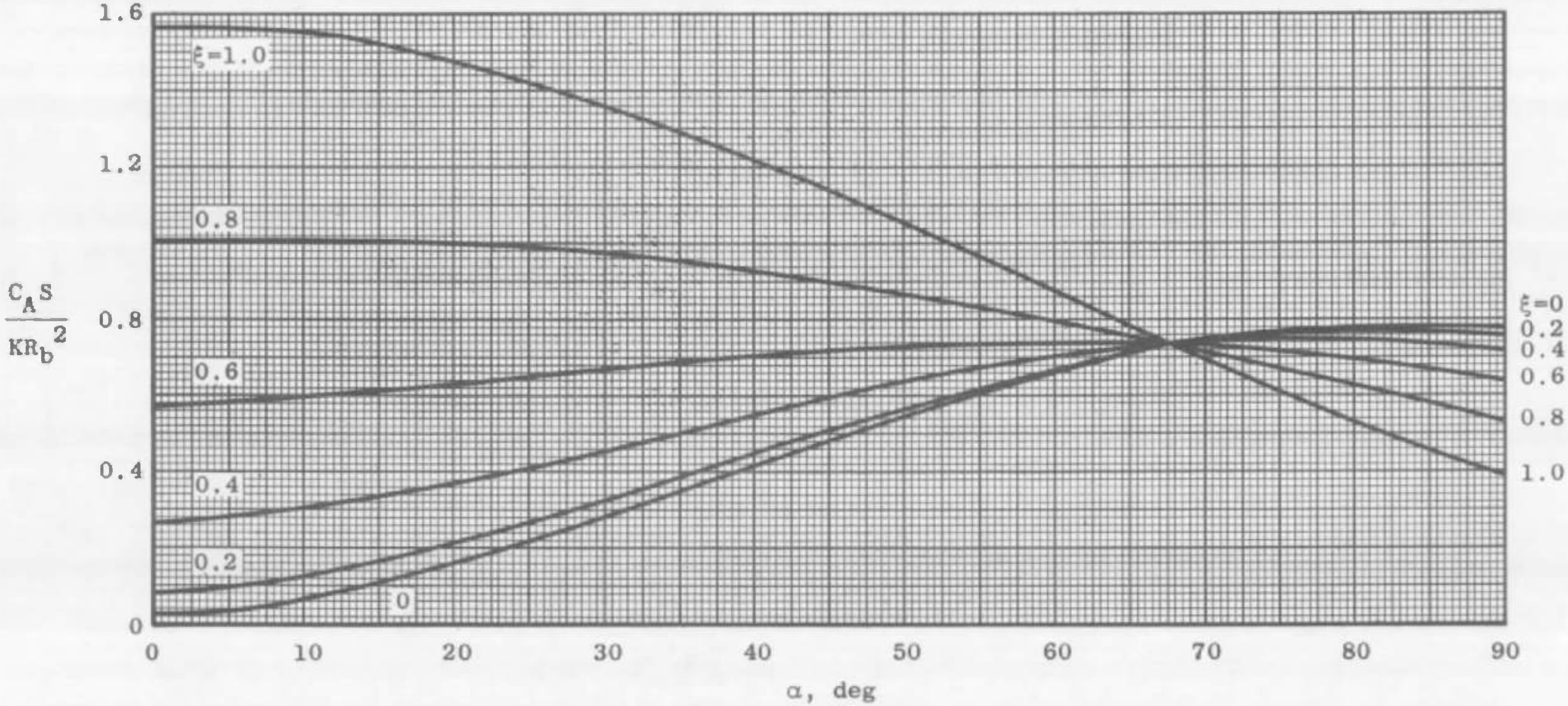
g. $\delta = 35$ deg

Fig. 2 Continued



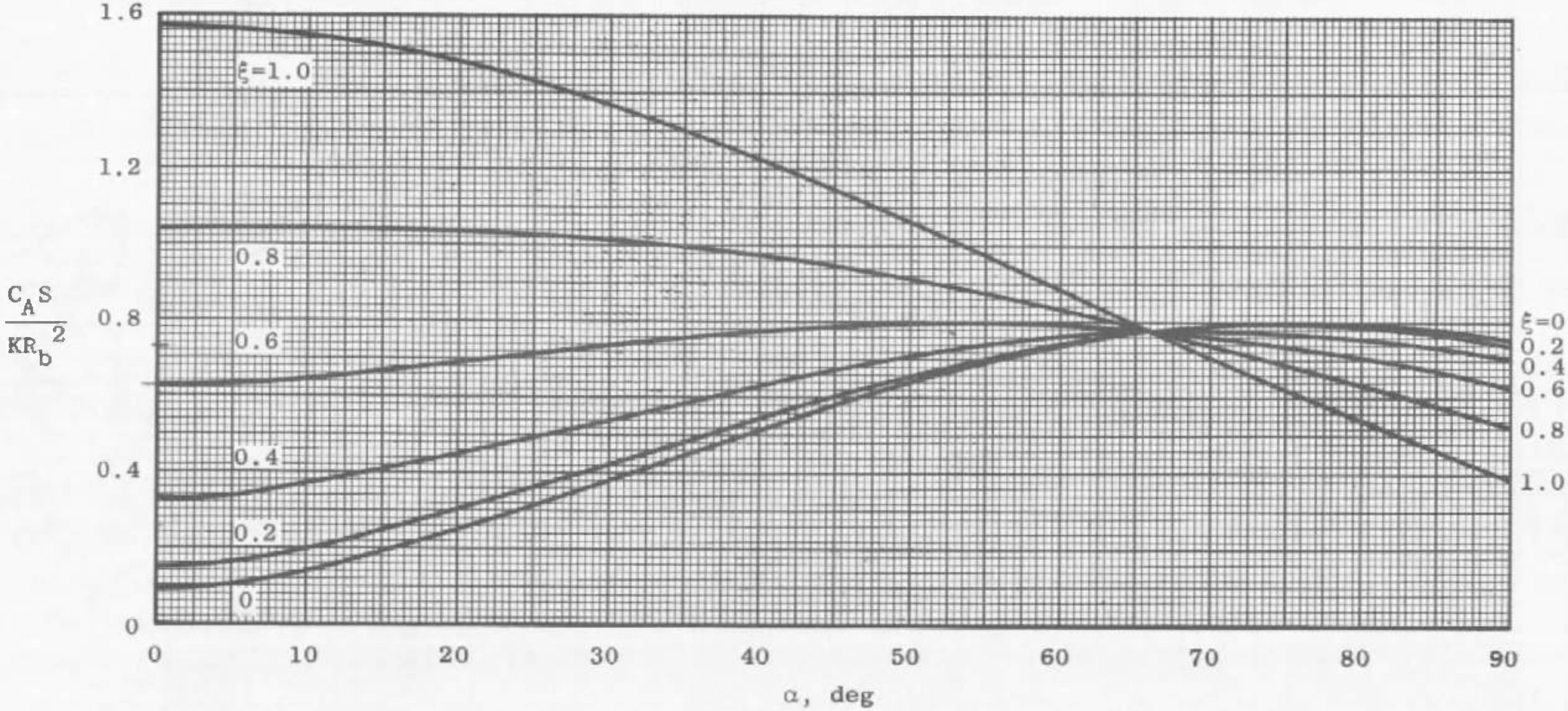
h. δ = 40 deg

Fig. 2 Concluded



a. δ = 5 deg

Fig. 3 Axial-Force Coefficient

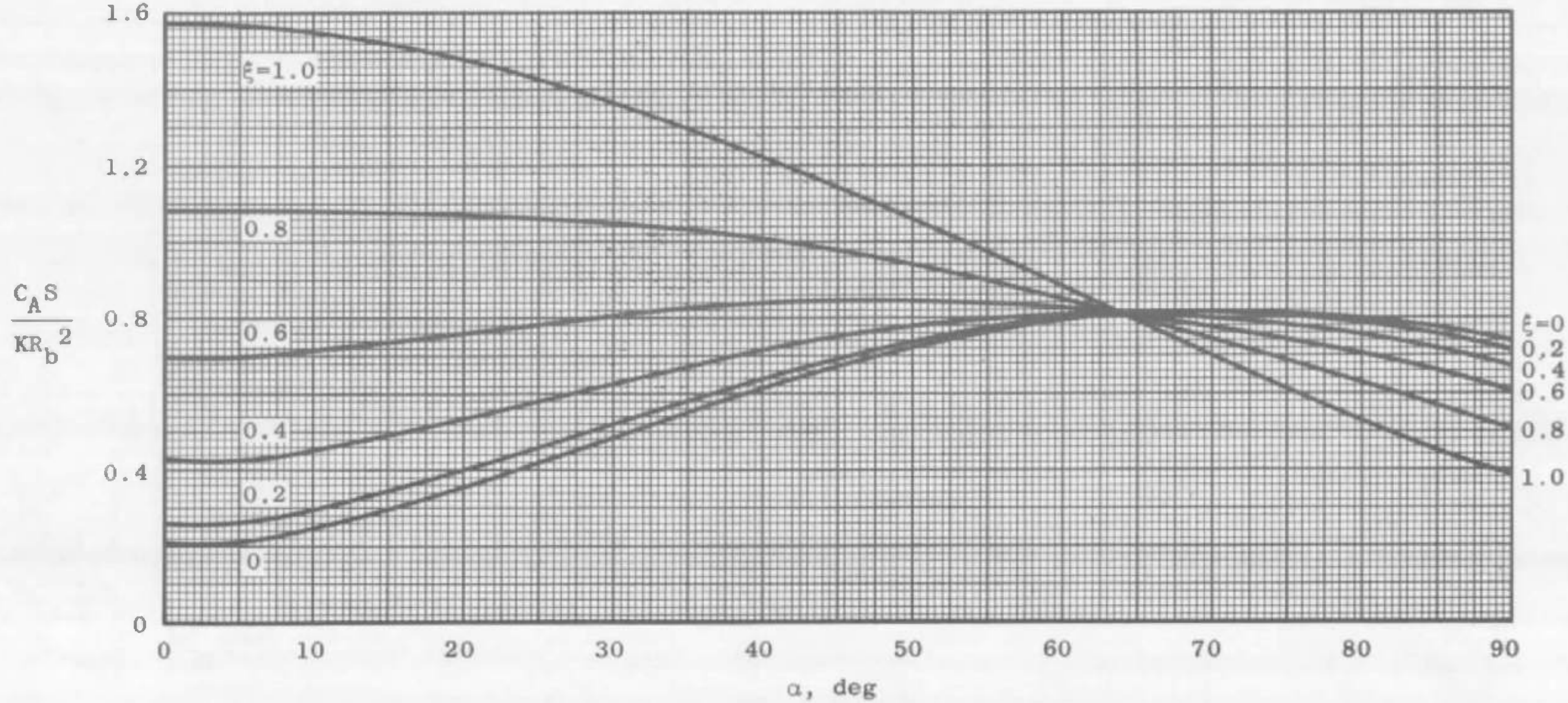


b. δ = 10 deg

Fig. 3 Continued

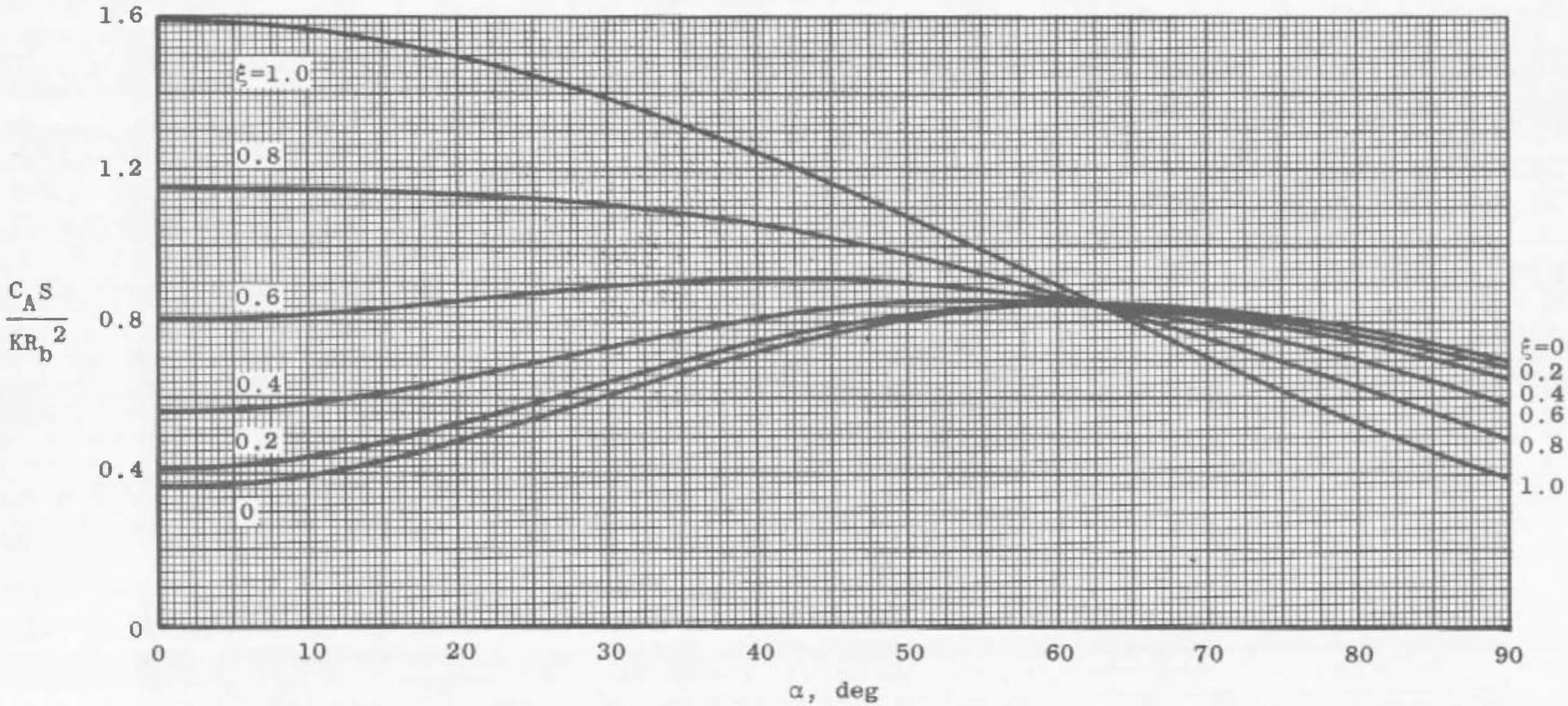
$S = \pi R_b^2$

$C_A (\pi/2)$



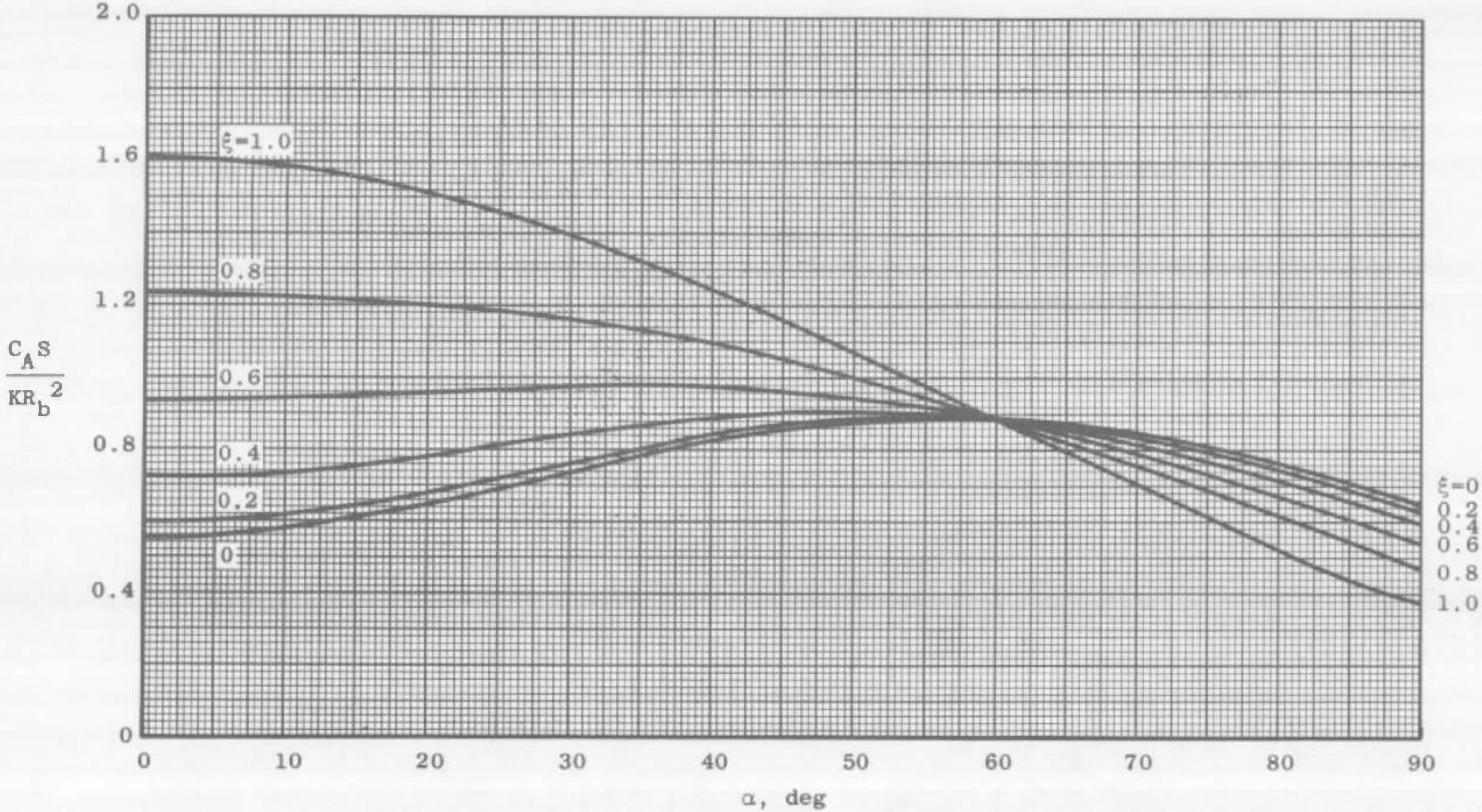
c. δ = 15 deg

Fig. 3 Continued



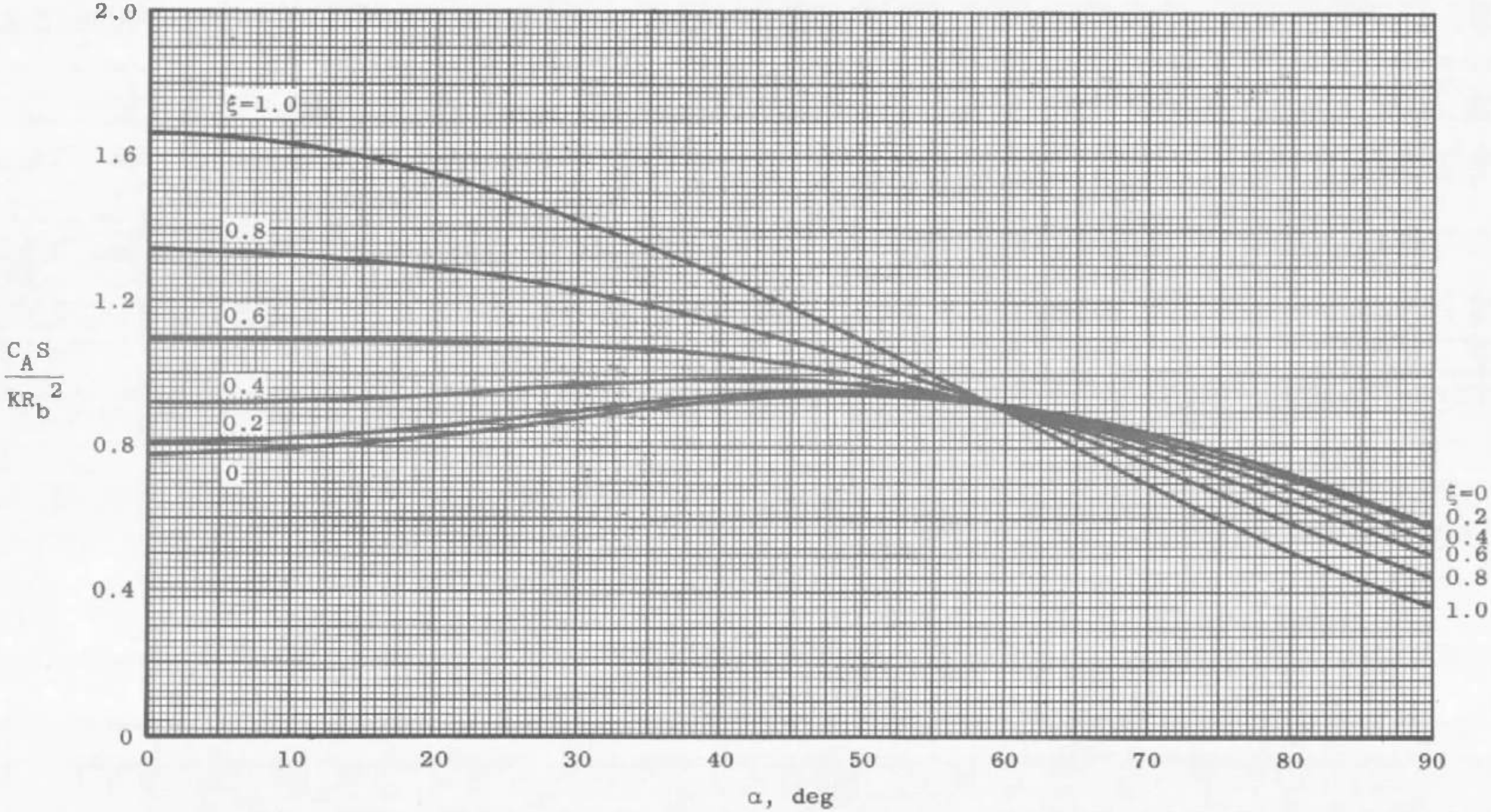
d. δ = 20 deg

Fig. 3 Continued



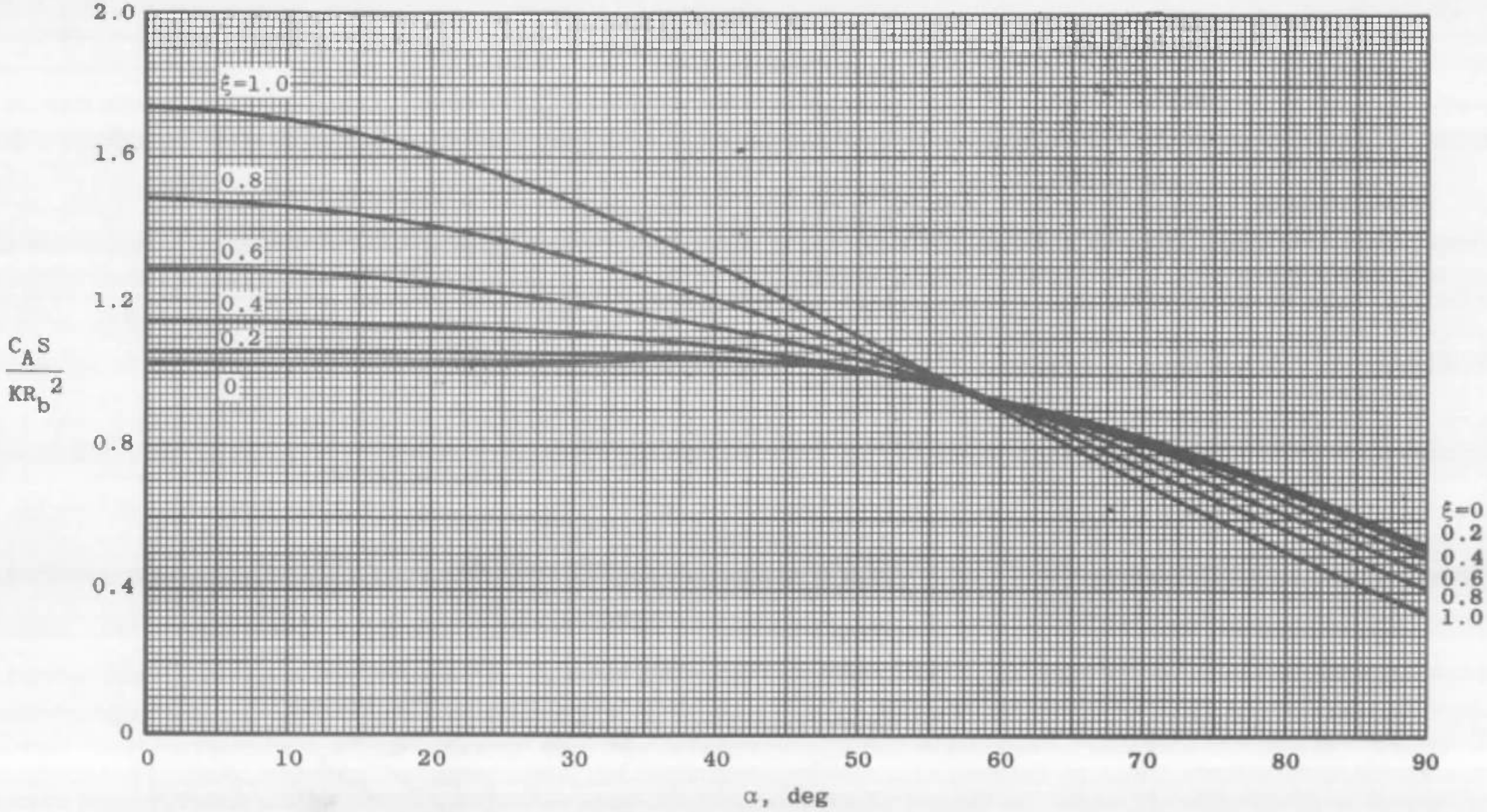
e. δ = 25 deg

Fig. 3 Continued



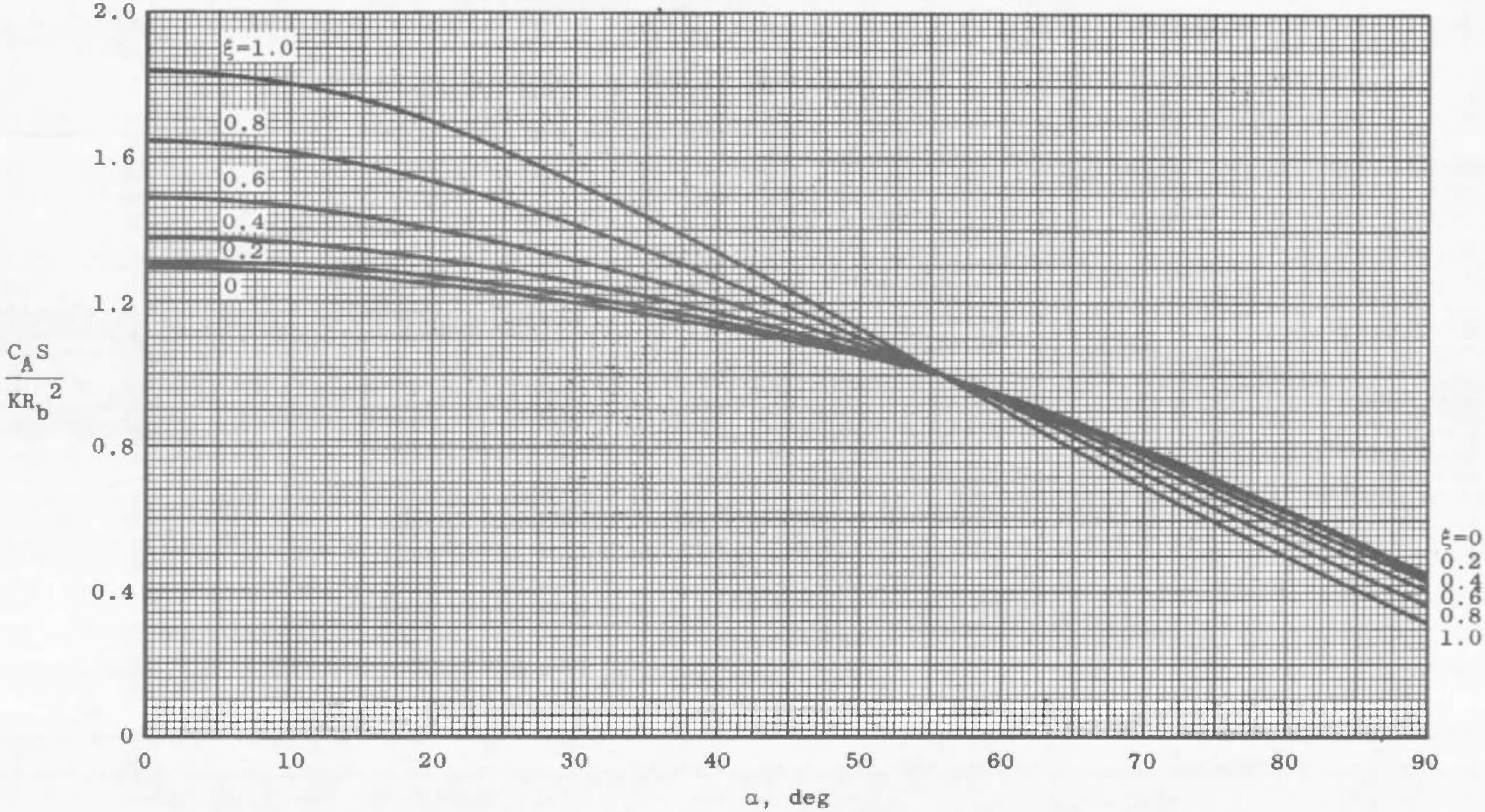
f. δ = 30 deg

Fig. 3 Continued



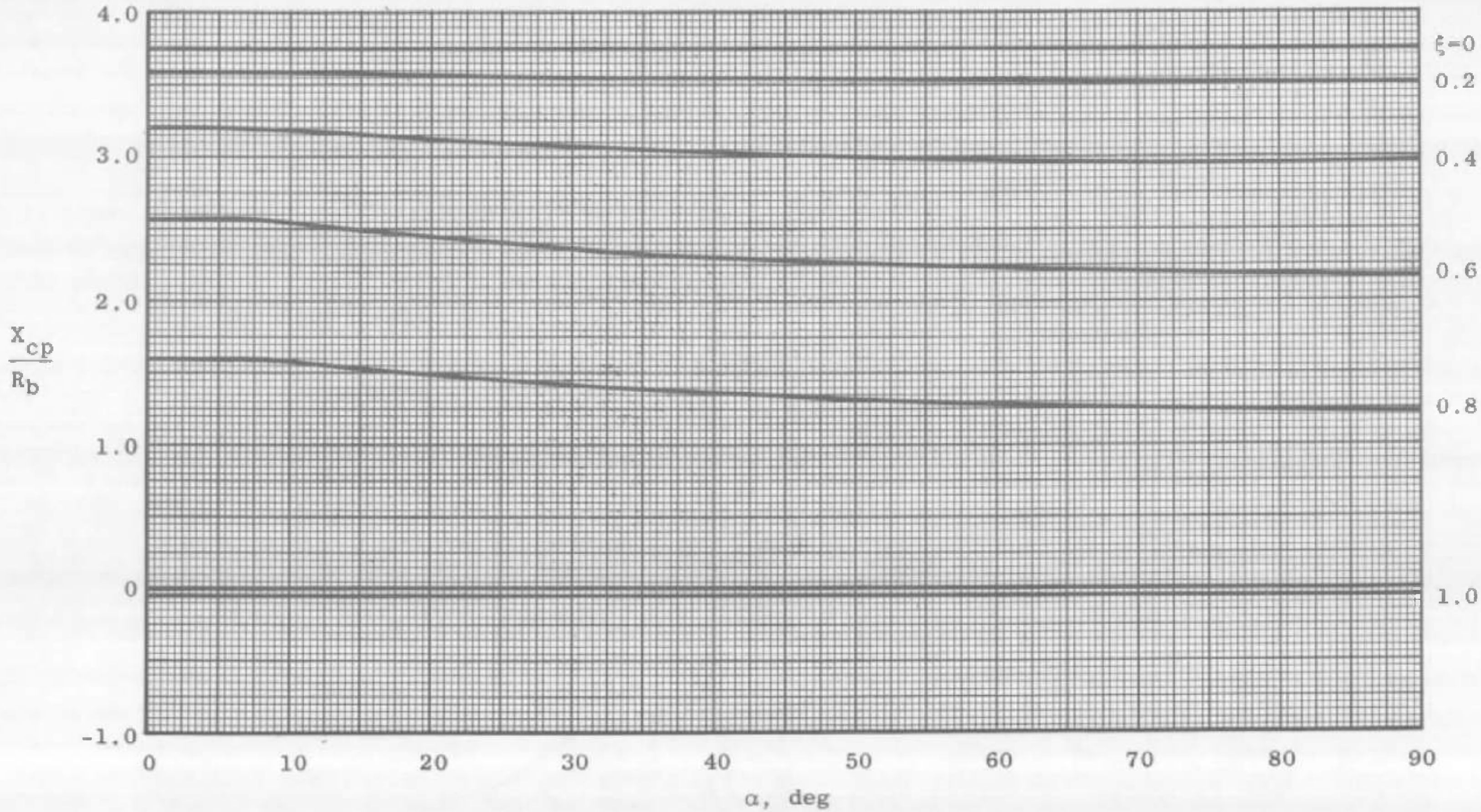
g. δ = 35 deg

Fig. 3 Continued



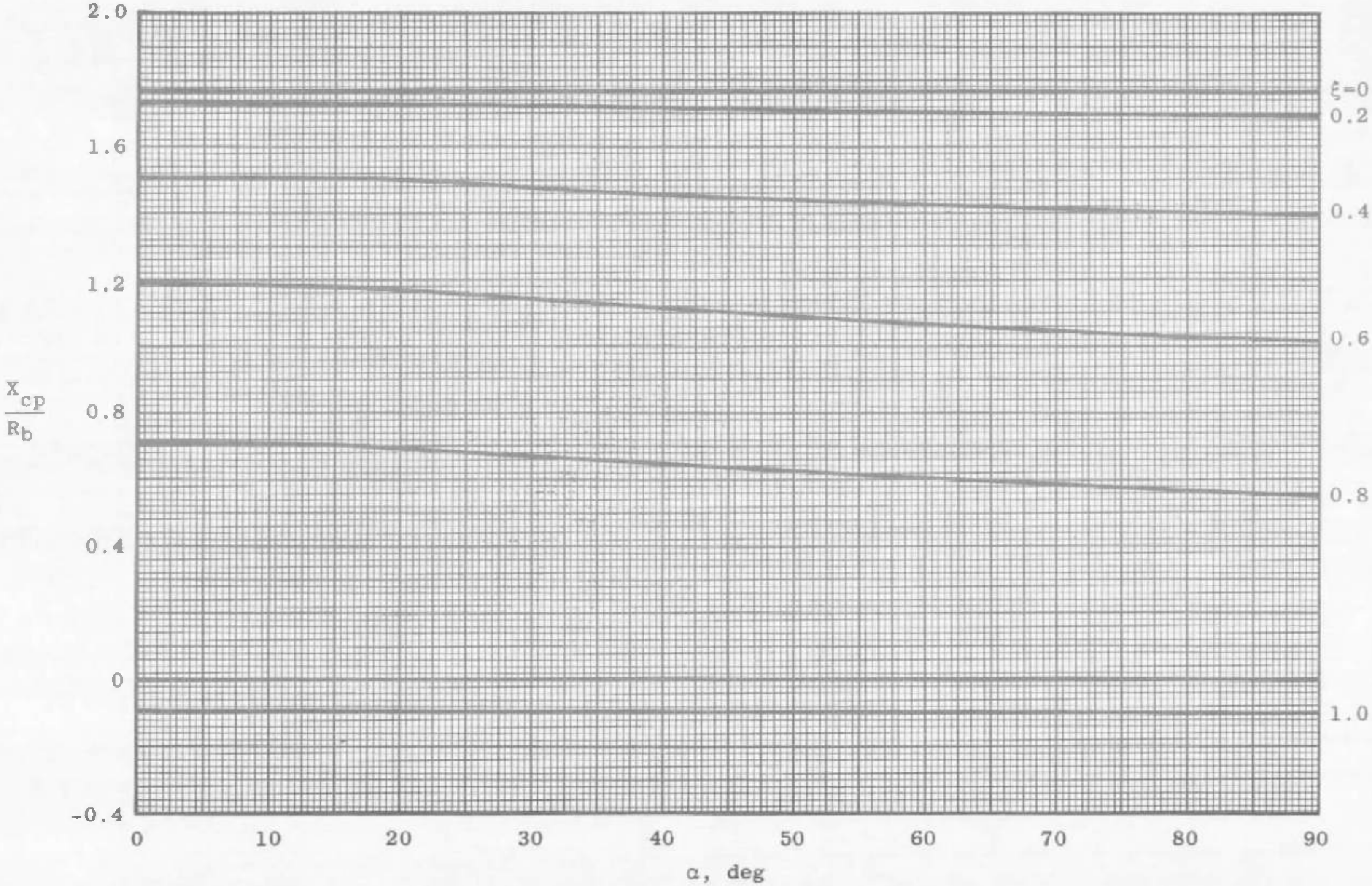
h. δ = 40 deg

Fig. 3 Concluded



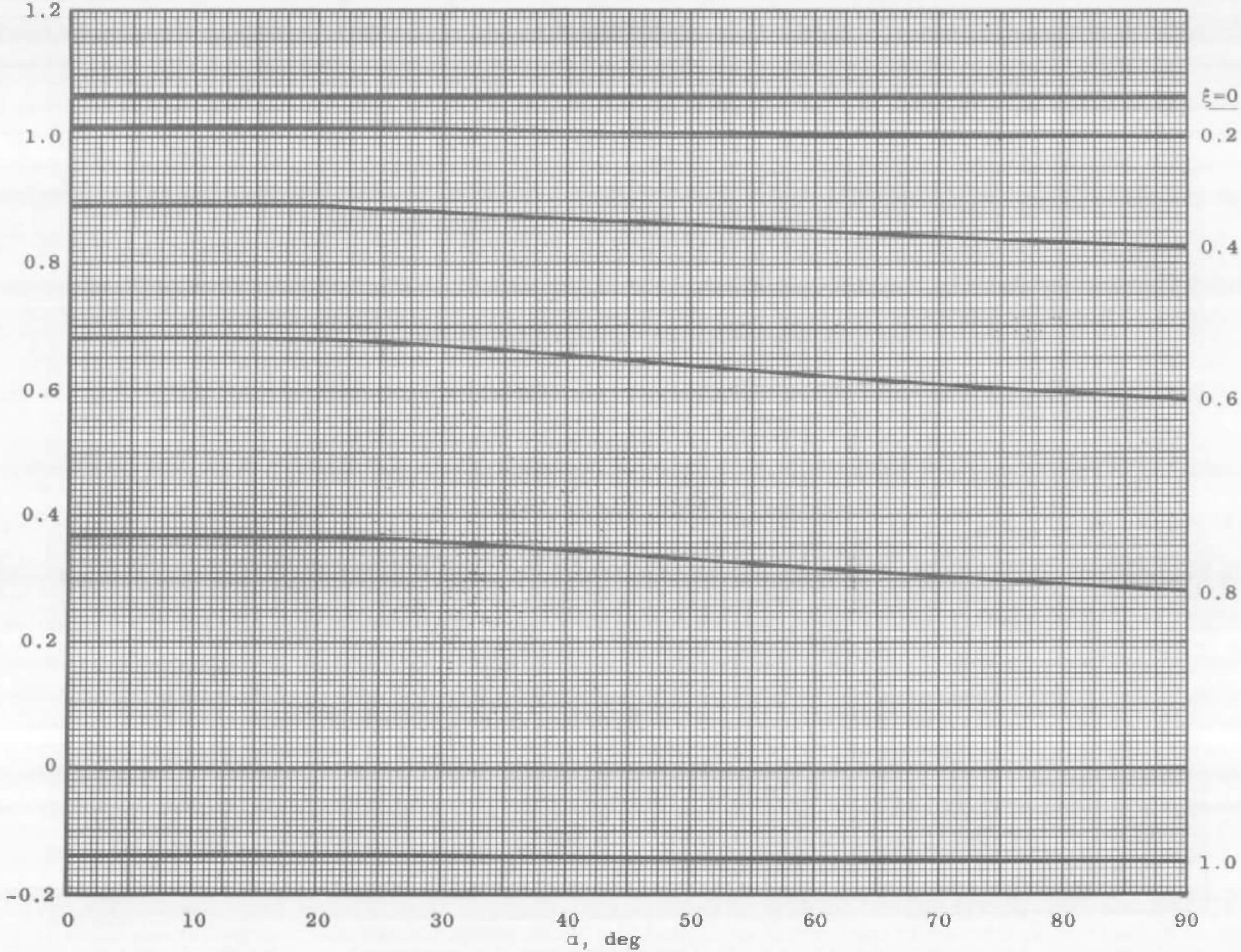
a. δ = 5 deg

Fig. 4 Center of Pressure



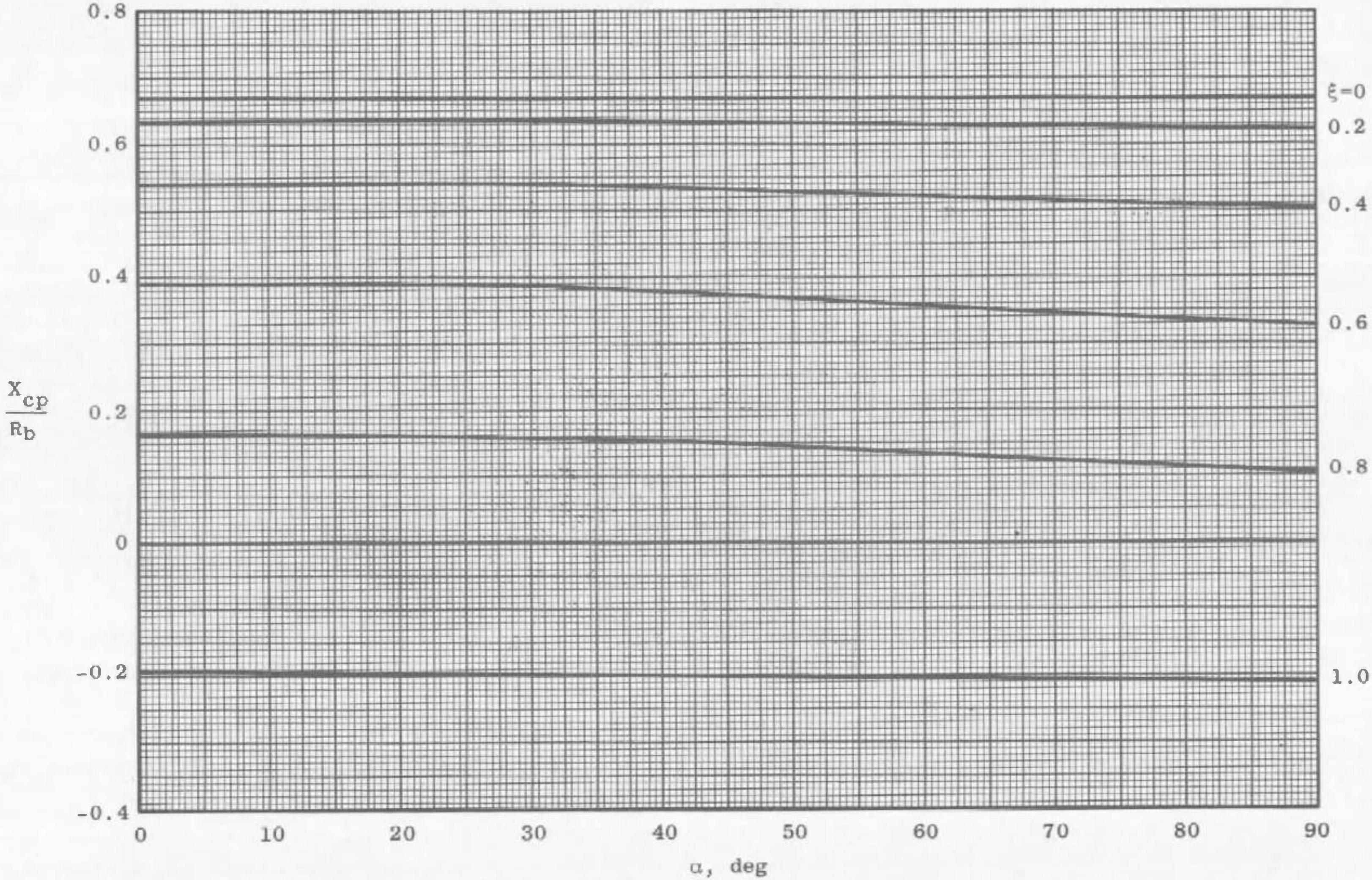
b. δ = 10 deg

Fig. 4 Continued



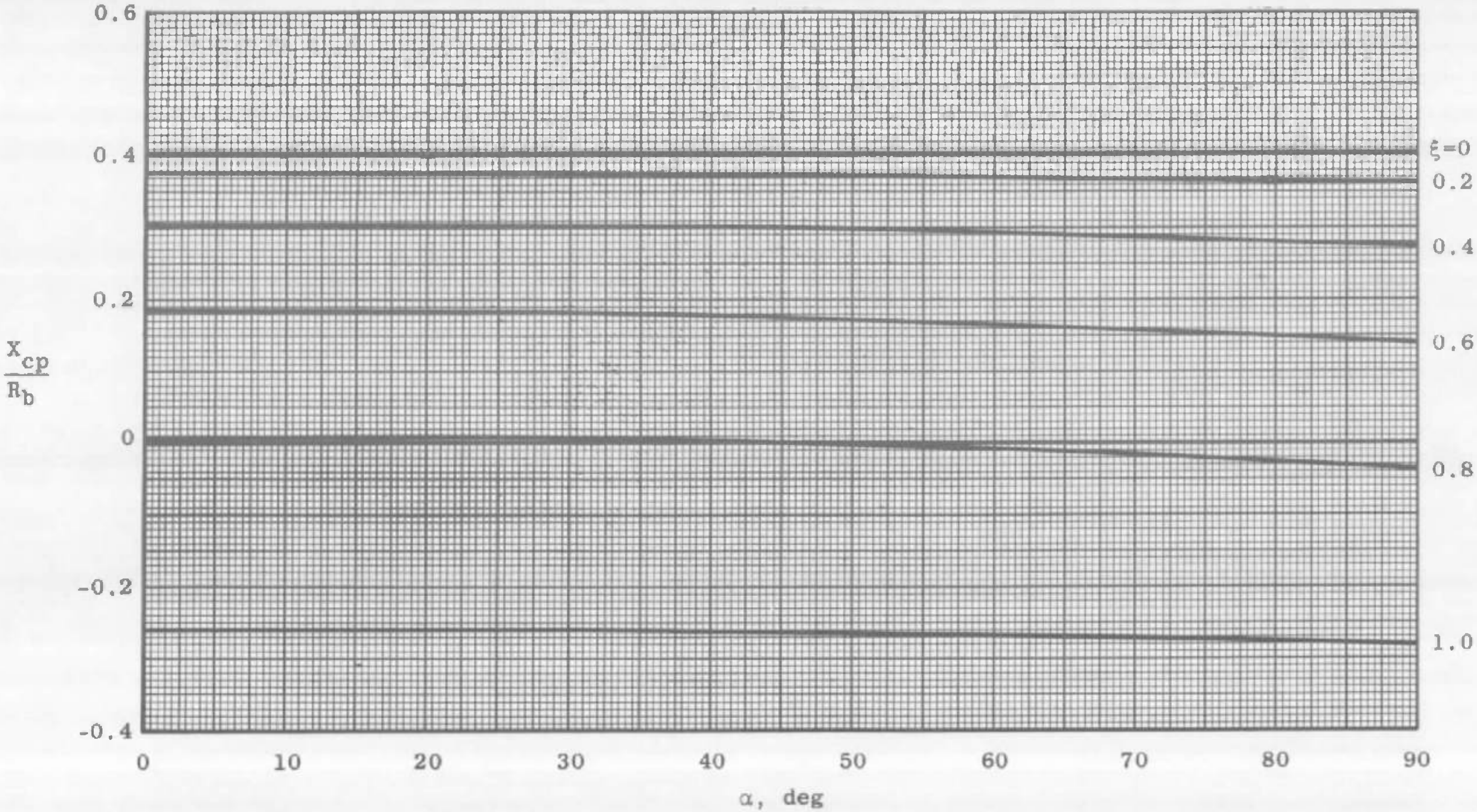
c. δ = 15 deg

Fig. 4 Continued



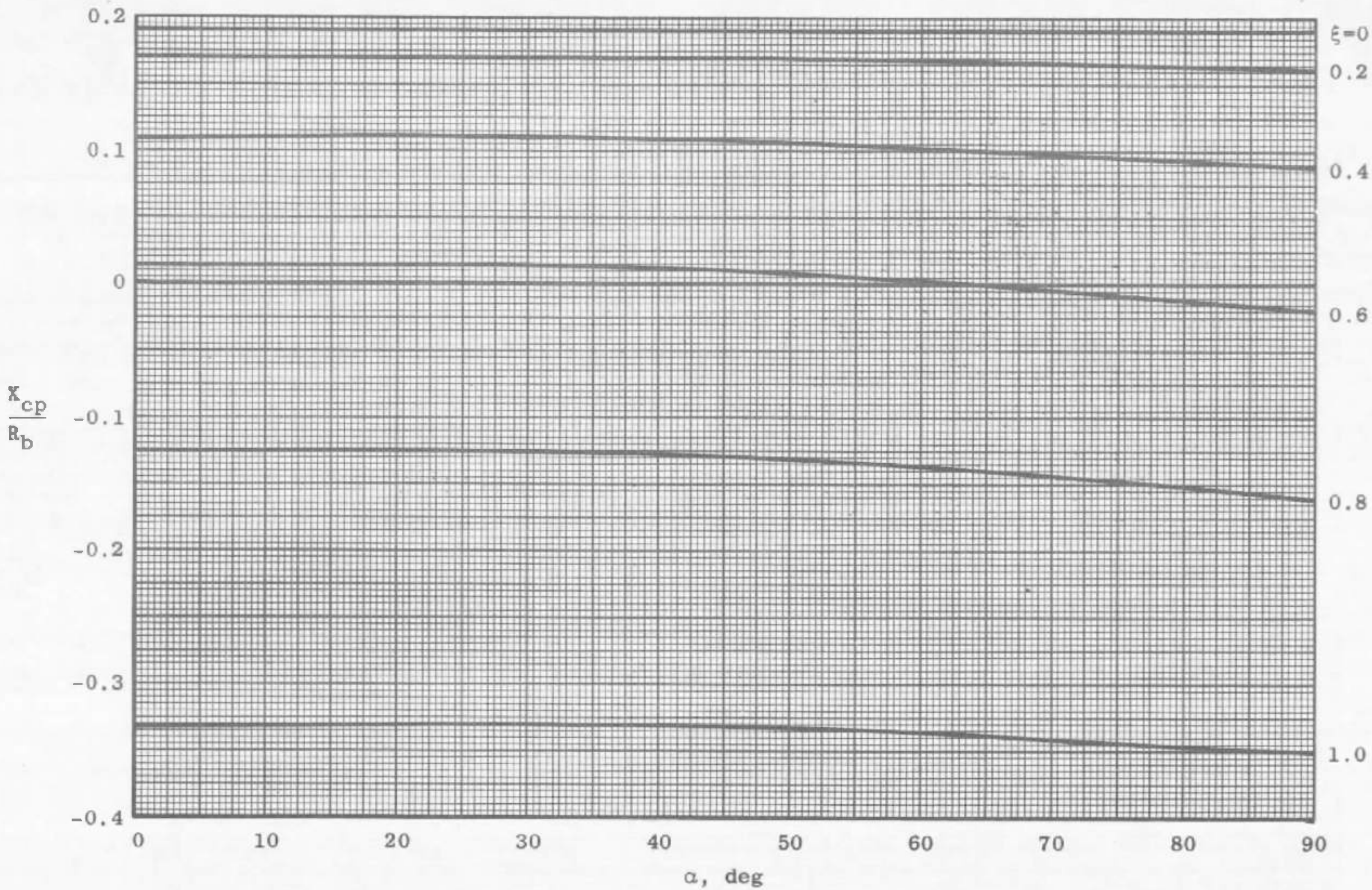
d. δ = 20 deg

Fig. 4 Continued



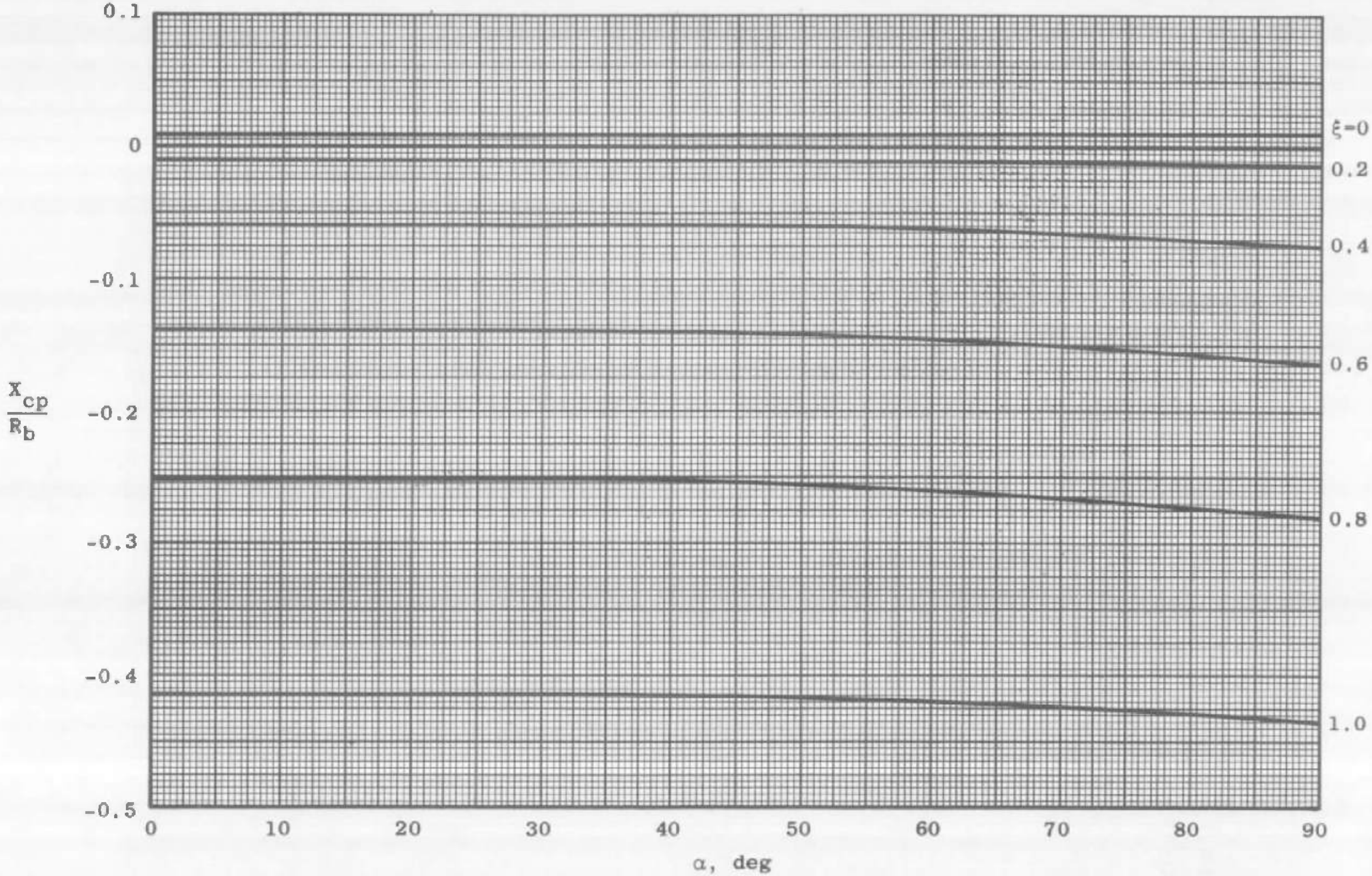
e. δ = 25 deg

Fig. 4 Continued



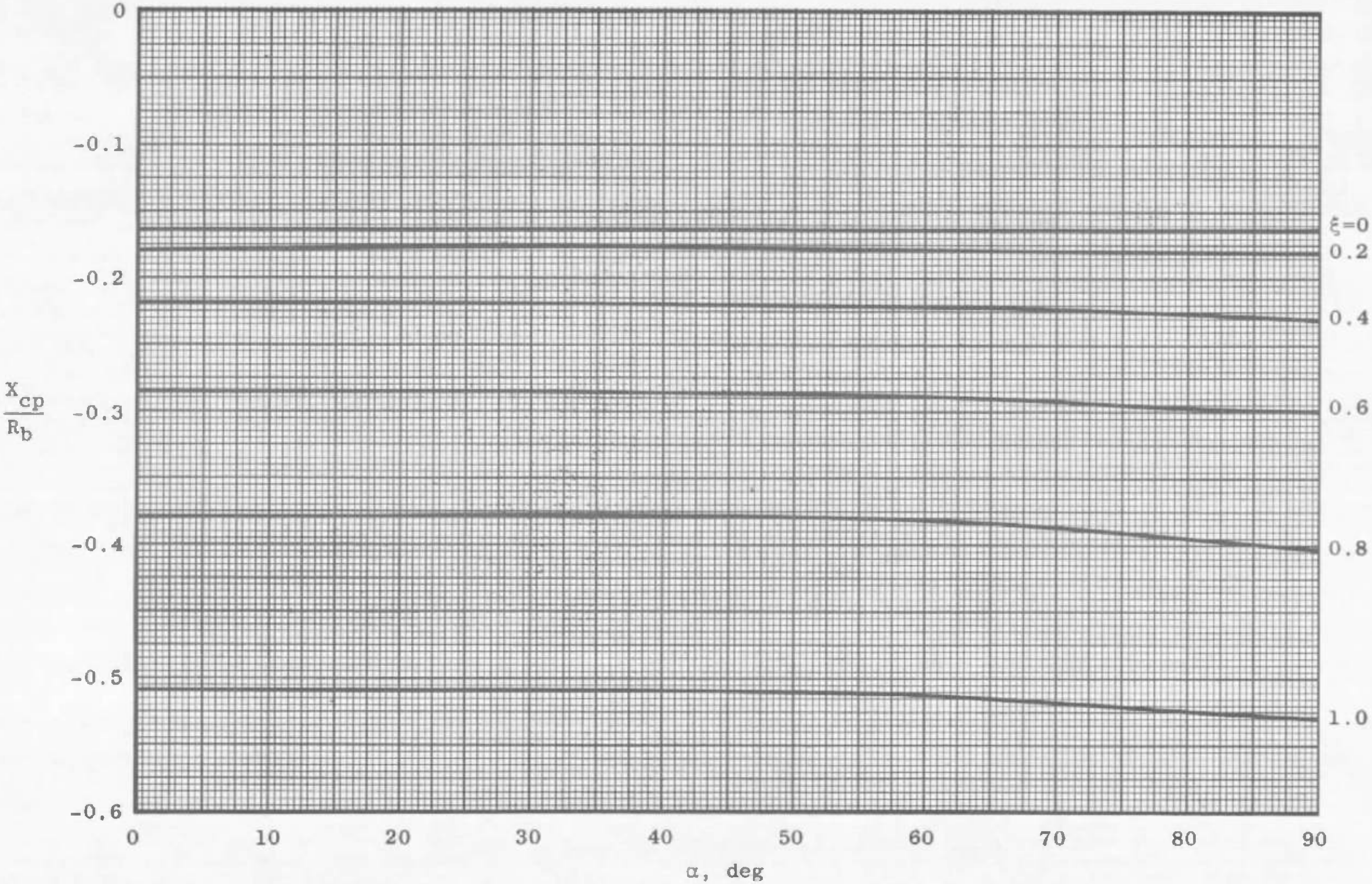
f. δ = 30 deg

Fig. 4 Continued



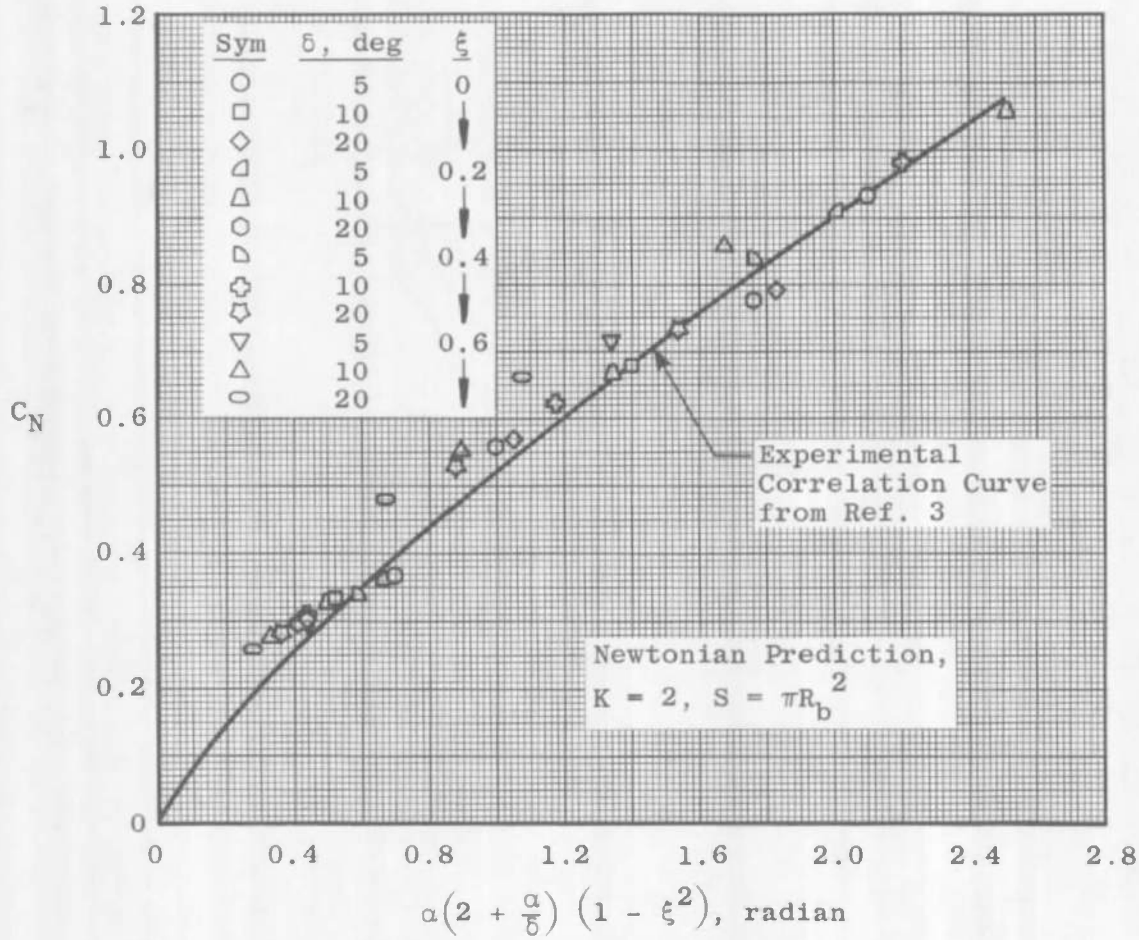
g. δ = 35 deg

Fig. 4 Continued



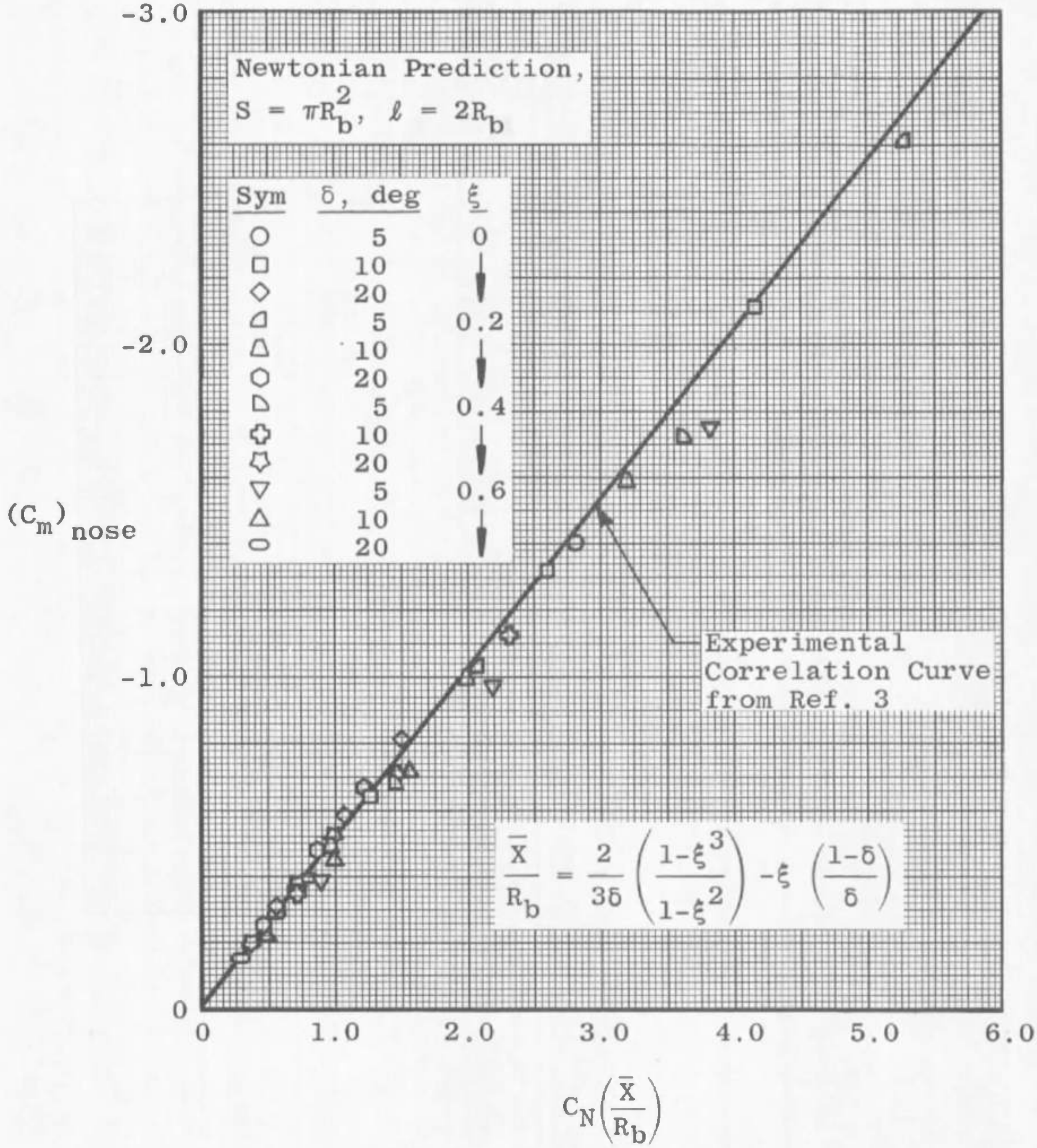
h. δ = 40 deg

Fig. 4 Concluded



a. Normal-Force Coefficient

Fig. 5 Comparison of Newtonian Predictions with Correlation Curves from Ref. 3



b. Pitching-Moment Coefficient

Fig. 5 Concluded

UNCLASSIFIED
Security Classification

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(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY (Corporate author) Arnold Engineering Development Center, ARO, Inc., Operating Contractor, Arnold Air Force Station, Tennessee	2a REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b GROUP N/A

3 REPORT TITLE

EQUATIONS AND CHARTS FOR THE EVALUATION OF FORCES ON SPHERICALLY BLUNTED CONES BY THE NEWTONIAN THEORY

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)

N/A

5 AUTHOR(S) (Last name, first name, initial)

Trimmer, L. L., ARO, Inc.

6 REPORT DATE	7a. TOTAL NO OF PAGES	7b NO OF REFS
April 1966	39	3

8a CONTRACT OR GRANT NO AF40(600)-1200	9a ORIGINATOR'S REPORT NUMBER(S) AEDC-TR-66-16
b. Program Element 65402234 c d	9b OTHER REPORT NO(S) (Any other numbers that may be assigned this report) N/A

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11 SUPPLEMENTARY NOTES	12 SPONSORING MILITARY ACTIVITY
N/A	Arnold Engineering Development Center, Air Force Systems Command, Arnold Air Force Station, Tennessee

13 ABSTRACT

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14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
spherically blunted cones						
force evaluation						
Newtonian theory						
hypersonic flow						
equations						
charts						

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